CSE 332 Autumn 2023
Lecture 13: Hashing

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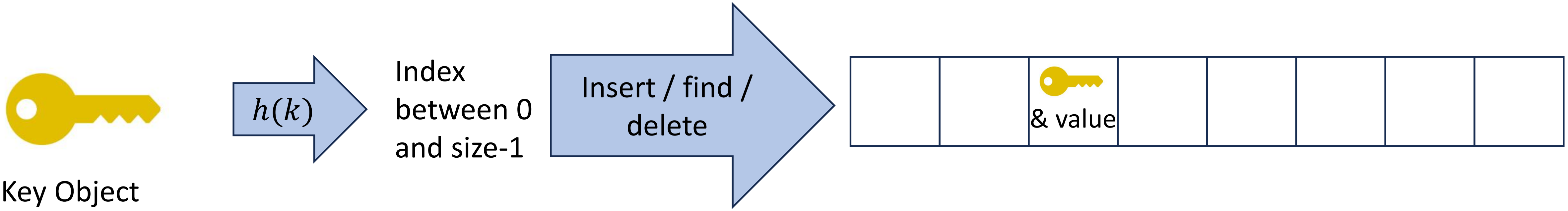
Dictionary Data Structures

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash Table (Worst case)	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Hash Table (Average)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should "scatter" the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



Key Object

$h(k)$

Index between 0 and size-1

Insert / find / delete

& value

What Influences Running time?

- How “spread out” our input keys are
    - How much do keys repeat
- Hash the function itself will take time
- Size of the table relative to the number things inserted
- How well our hash function scatters the keys
- What do we do when two things hash to the same spot

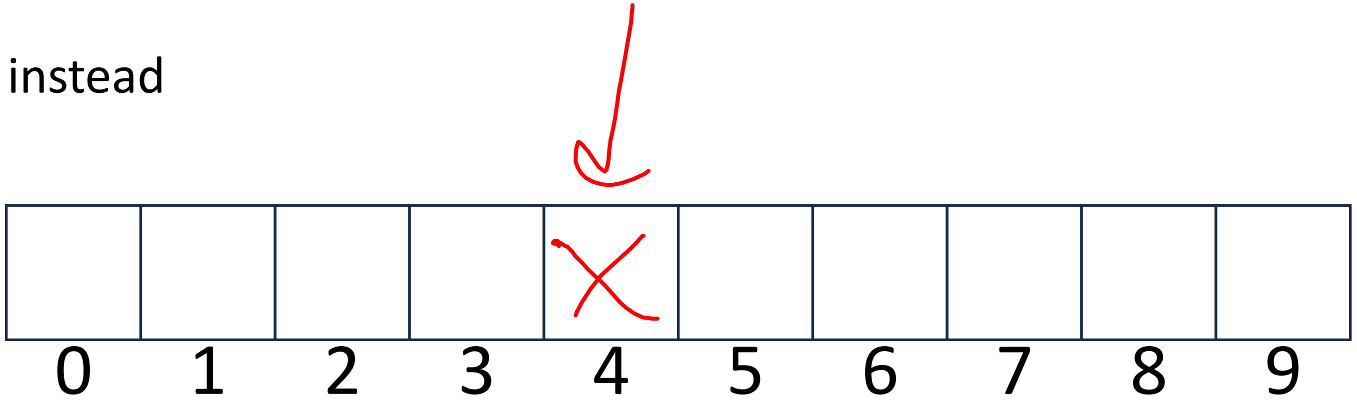
Properties of a "Good" Hash

- Definition: A hash function maps objects to integers

- Should be very efficient
  - Calculating the hash should be negligible
- Should randomly scatter objects
  - Objects that are similar to each other should be likely to end up far away
- Should use the entire table
  - There should not be any indices in the table that nothing can hash to
  - Picking a table size that is prime helps with this
- Should use things needed to "identify" the object
  - Use only fields you would check for a .equals method be included in calculating the hash
  - More fields typically leads to fewer collisions, but less efficient calculation

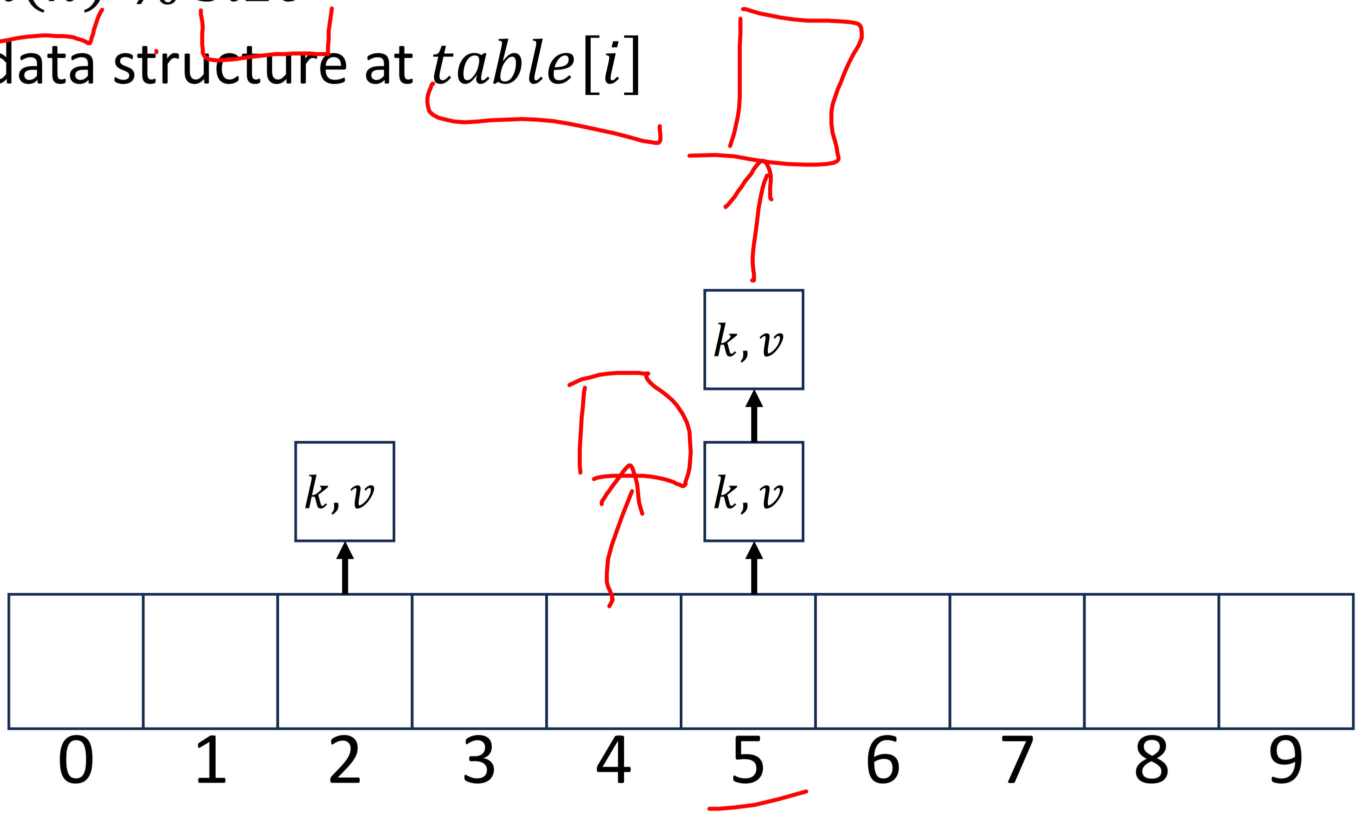
Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
    - Separate Chaining
        - Use a secondary data structure to contain the items
            - E.g. each index in the hash table is itself a linked list
    - Open Addressing
        - Use a different spot in the table instead
            - Linear Probing
            - Quadratic Probing
            - Double Hashing



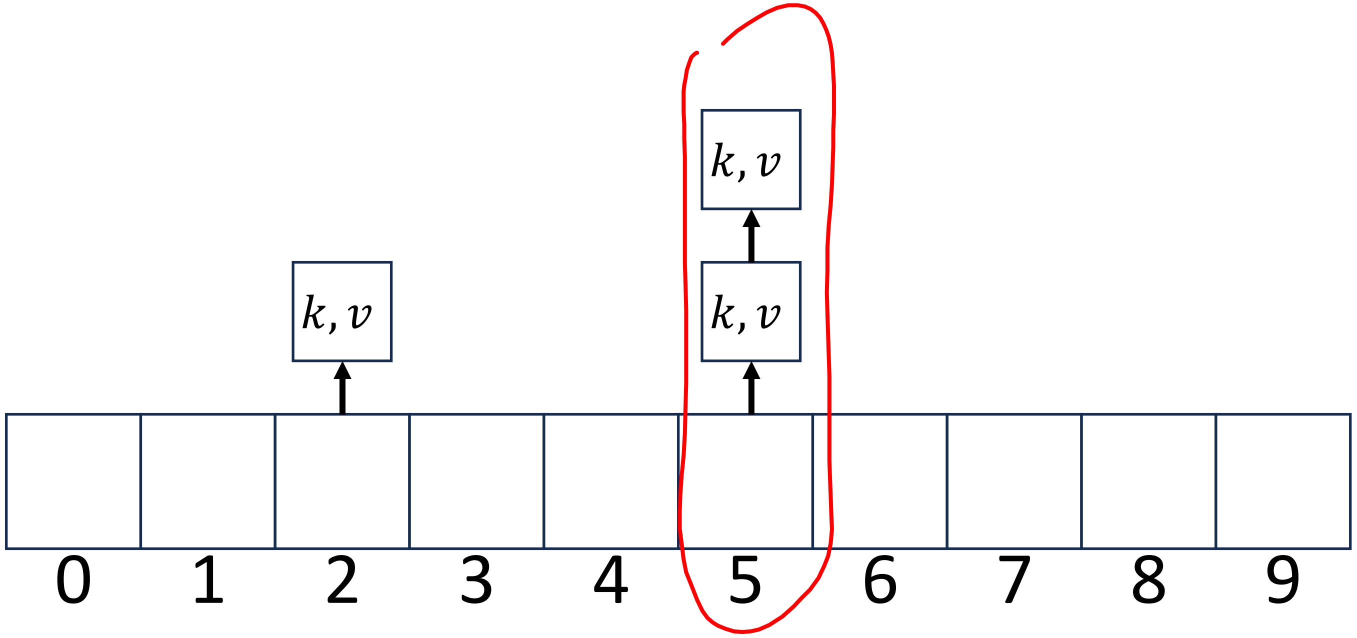
Separate Chaining Insert

- To insert k, v:
  - Compute the index using $i = h(k) \% size$
  - Add the key-value pair to the data structure at $table[i]$



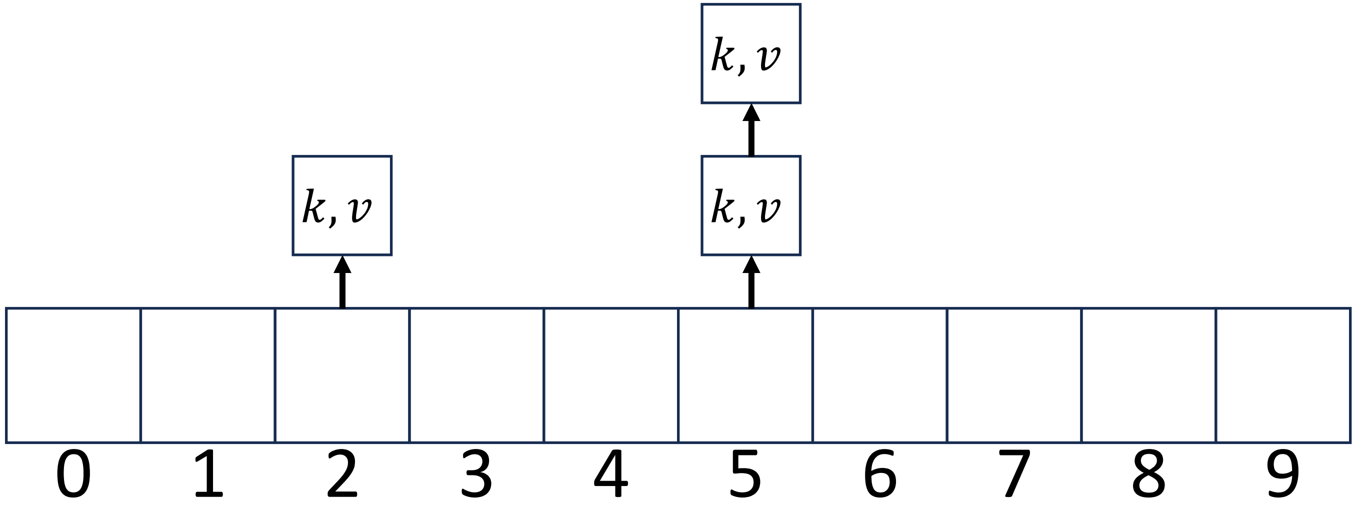
Separate Chaining Find

- To find k:
  - Compute the index using $i = h(k)\ \%\ size$
  - Call find with the key on the data structure at $table[i]$



Separate Chaining Delete

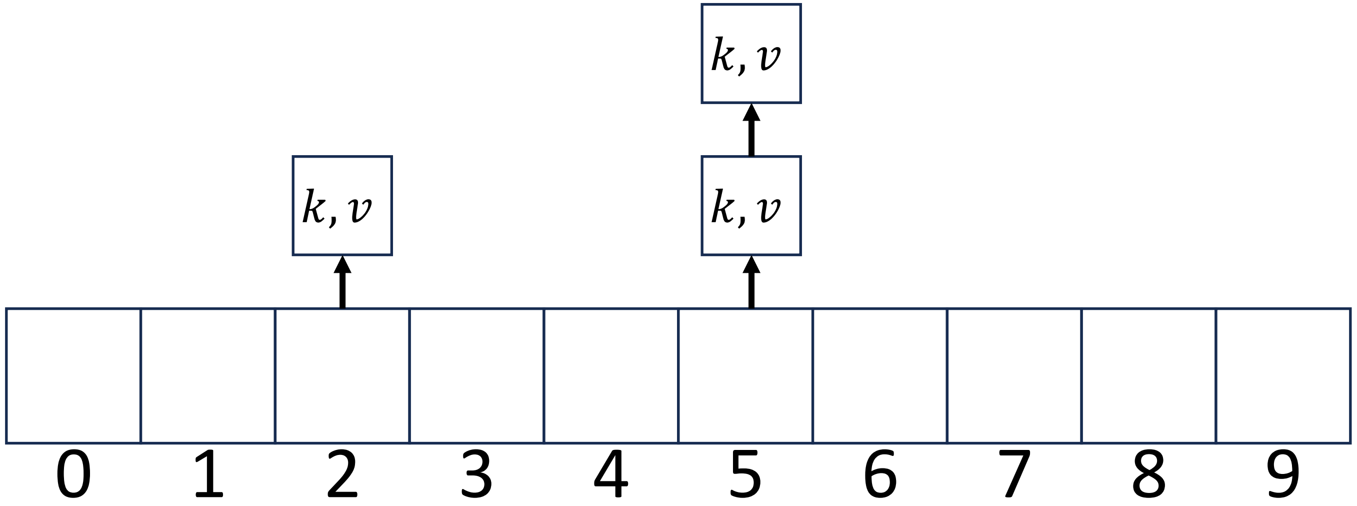
- To delete k:
    - Compute the index using $i = h(k) \ \% \ size$
    - Call delete with the key on the data structure at $table[i]$



Formal Running Time Analysis

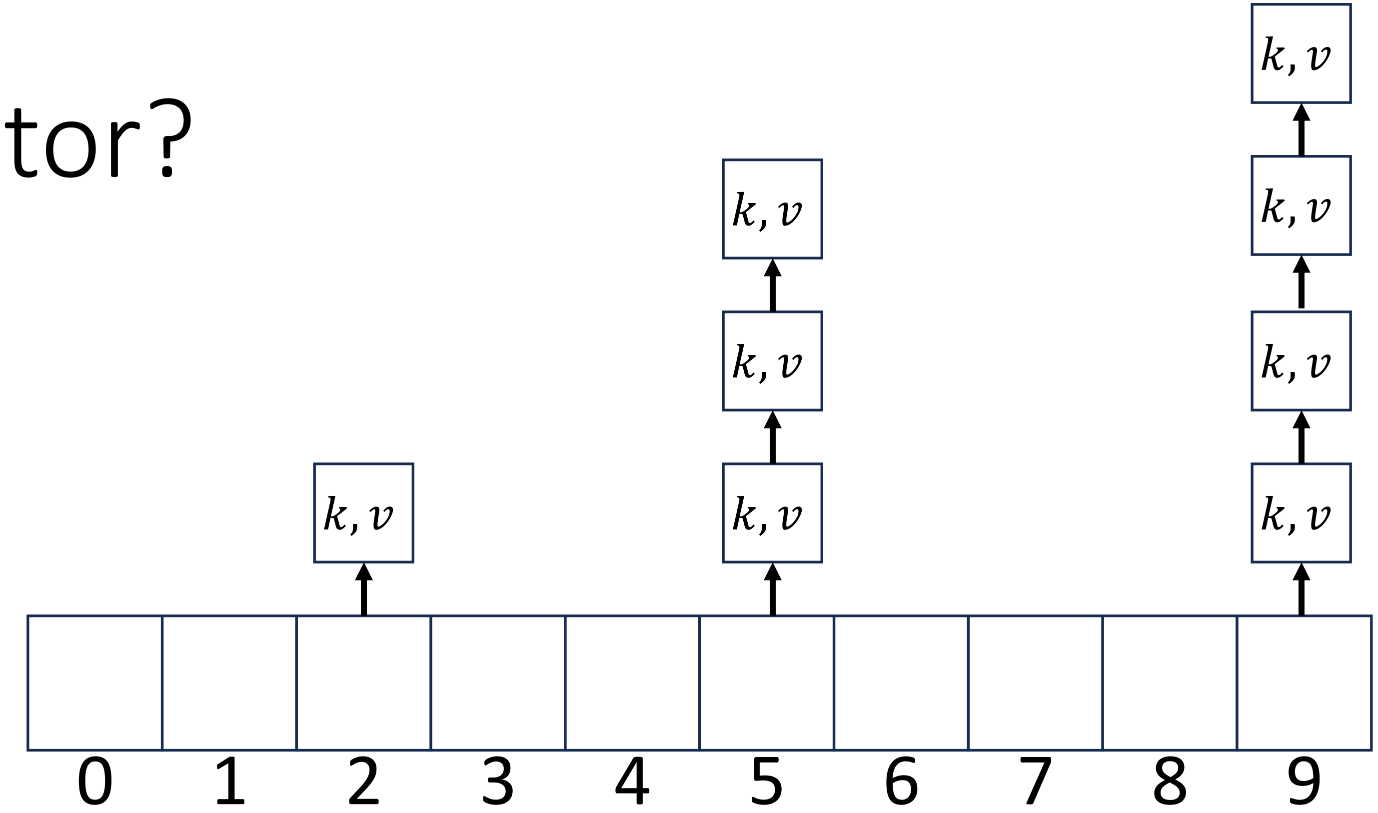
- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{size}$
- Assume we have a has table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - In general: an unsuccessful find will be linear in the length of the list we hash to
    - λ
  - What is the expected number of comparisons needed in a successful find?
    - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - We need to make λ constant
  - Make the size of the hash proportional to the number of things in it

Load Factor?



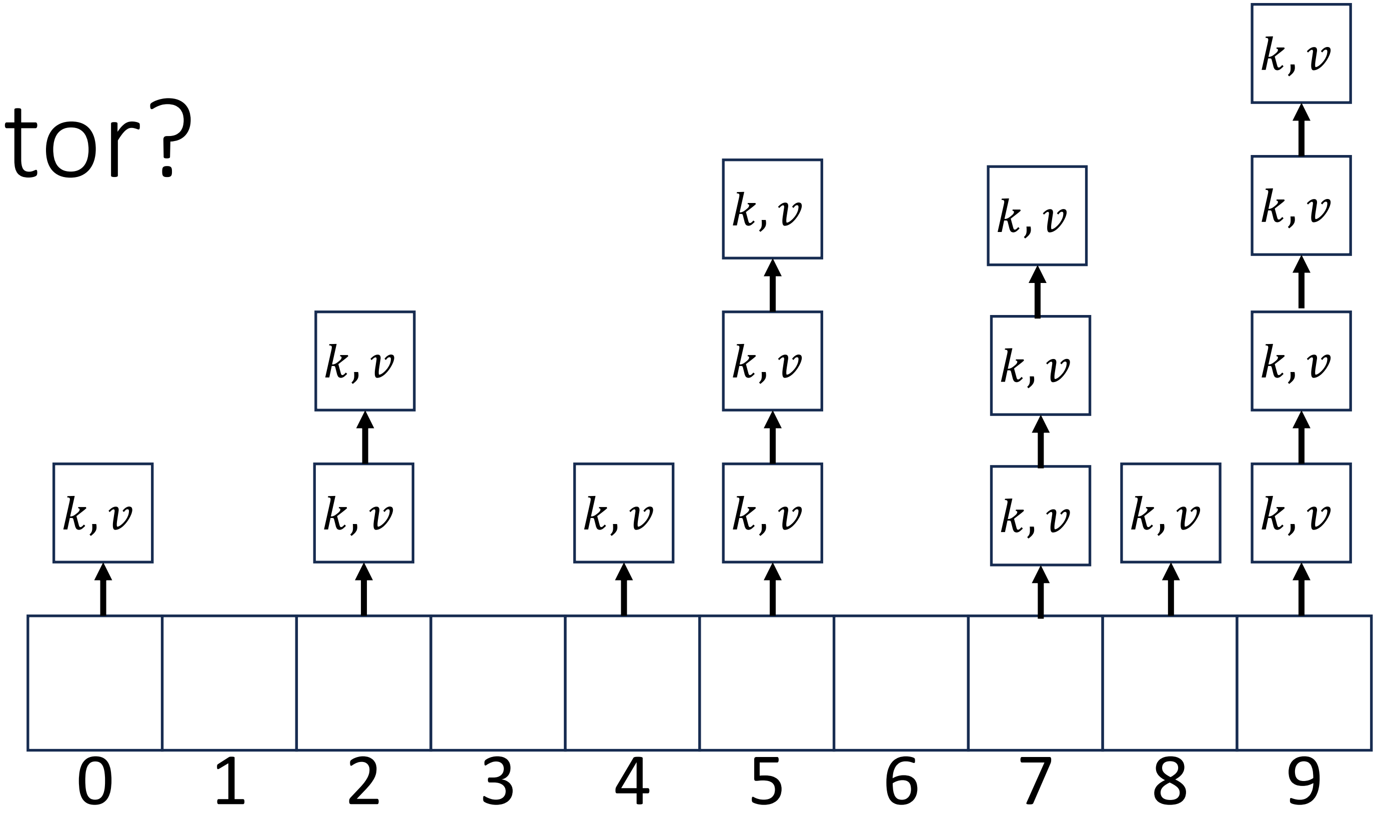
$\frac{3}{10} = 0.3$

Load Factor?



$\frac{8}{10} = 0.8$

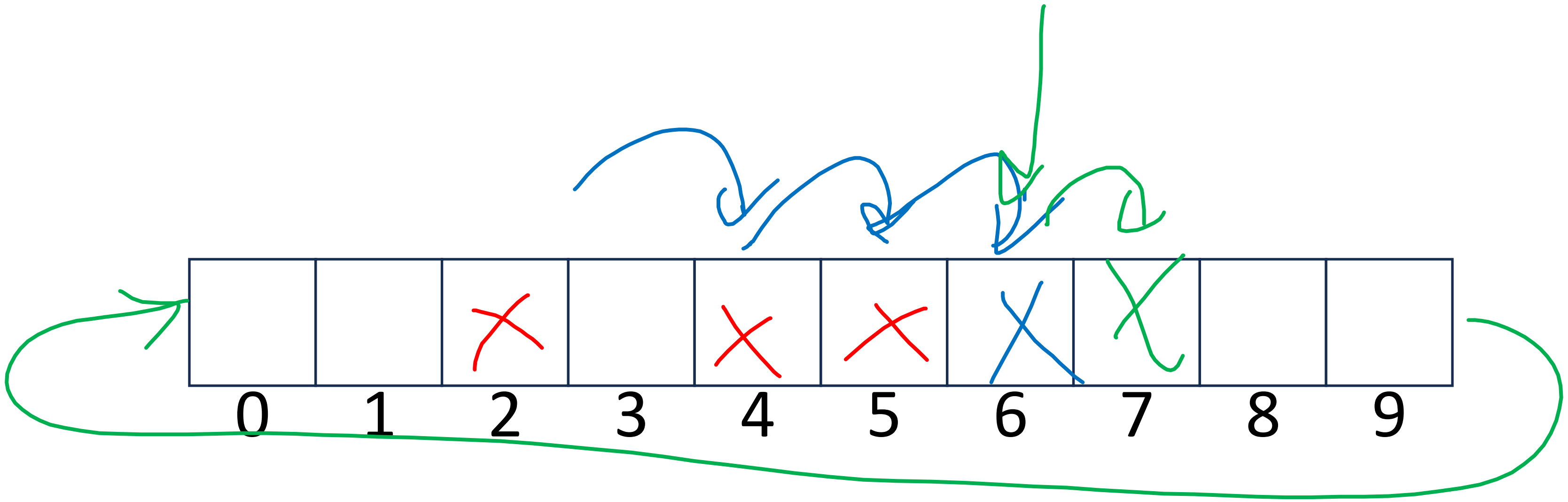
Load Factor?



$\frac{15}{10} = 1.5$

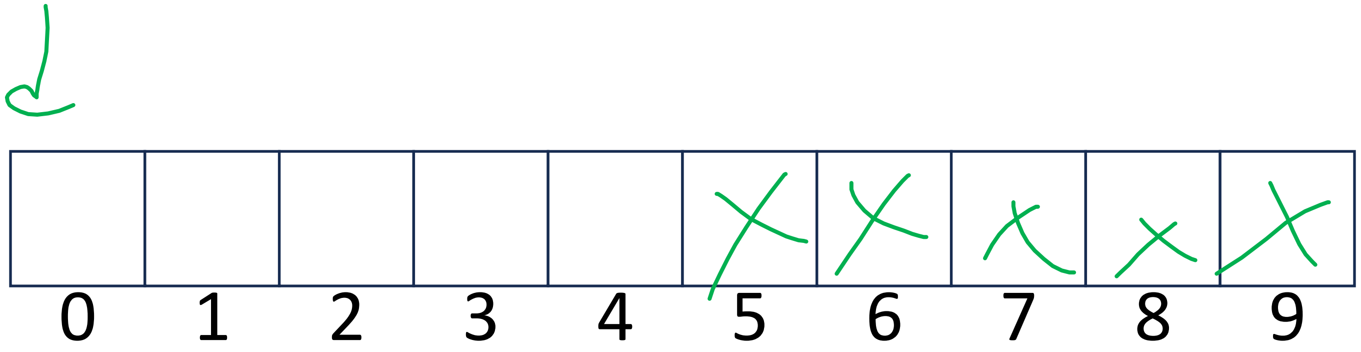
Collision Resolution: Linear Probing

- When there's a collision, use the next open space in the table



Linear Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k) \% size$
  - If $table[i]$ is occupied then try $(i+1)\% size$
  - If that is occupied try $(i+2)\% size$
  - If that is occupied try $(i+3)\% size$
  - …



Linear Probing: Find

- $i = h(k)\%size$
  - If i has the key or it's empty, then we're done
  - Otherwise:
    - Check $(i+1)\%size$ if it's there, done else
    - Check $(i+2)\%size$ if it's there, done else
    - Check $(i+3)\%size$
    - …
    - Until we hit an empty cell

"wants"

a lex

small

$\lambda << 1$

Linear Probing: Find

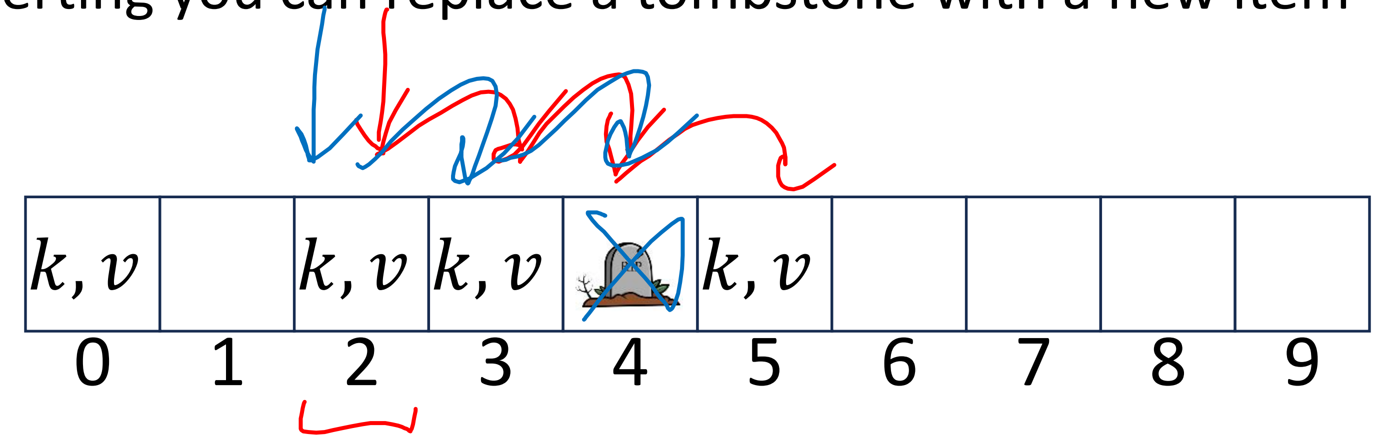
- To find key k
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied and does not contain k then look at $(i + 1)\ \%\ size$
  - If that is occupied and does not contain k then look at $(i + 2)\ \%\ size$
  - If that is occupied and does not contain k then look at $(i + 3)\ \%\ size$
  - Repeat until you either find k or else you reach an empty cell in the table

Linear Probing: Delete

- Problem: don’t want to leave an empty space when deleting
- Option 1: when we delete, move the “last thing” with a matching hash to that location
- Option 2: “tombstone” deletion. When deleting something, leave a special marker to indicate something used to be there
-

Linear Probing: Delete

- Option 1: Find the last thing with a matching hash, move that into the spot you deleted from
- Option 2: Called "tombstone" deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

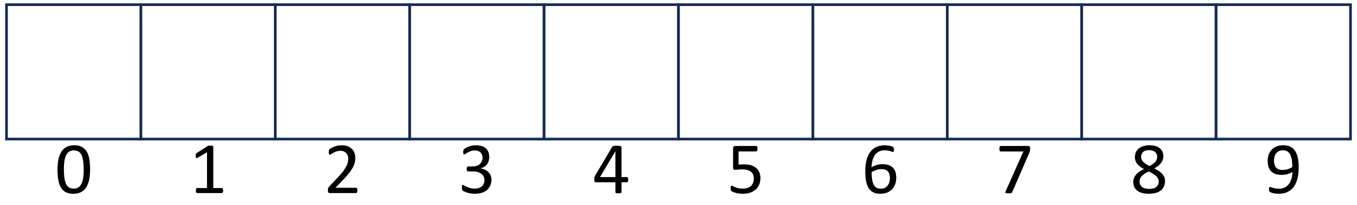


Downsides of Linear Probing

- What happens when λ approaches 1?
  - Runnings times get longer and longer
- What happens when λ exceeds 1?
  - Run out of space
- We need a really small λ

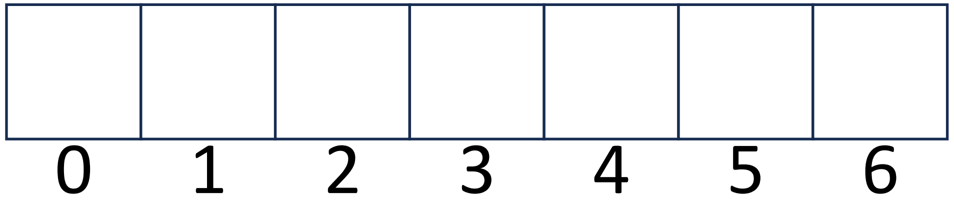
Quadratic Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $(i + 1^2)\%\ size$
  - If that is occupied try $(i + 2^2)\%\ size$
  - If that is occupied try $(i + 3^2)\%\ size$
  - If that is occupied try $(i + 4^2)\%\ size$
  - …



Quadratic Probing: Example

- Insert:
    - 76
    - 40
    - 48
    - 5
    - 55
    - 47



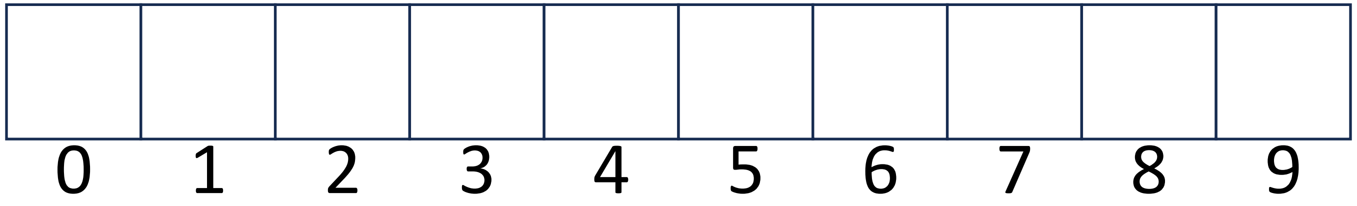
Using Quadratic Probing

- If you probe $tablesize$ times, you start repeating the same indices
- If $tablesize$ is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most $tablesize/2$ probes

- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

Double Hashing: Insert Procedure

- Given h and g are both good hash functions
- To insert k, v
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $\left(i + g(k)\right)\ \%\ size$
  - If that is occupied try $\left(i + 2 \cdot g(k)\right)\%\ size$
  - If that is occupied try $\left(i + 3 \cdot g(k)\right)\%\ size$
  - If that is occupied try $\left(i + 4 \cdot g(k)\right)\%\ size$
  - …



Rehashing

- If your load factor λ gets too large, copy everything over to a larger hash table
    - To do this: make a new array with a new hash function
    - Re-insert all items into the new hash table with the new hash function
    - New hash table should be “roughly” double the size (but probably still want it to be prime)