Section 8: Minimax & Alpha Beta Pruning

CSE 332 19Au

University of Washington

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A Game algorithm: Minimax

Some backgrounds on the game

- Let's assume that our opponent plays optimally
- Let's assume that we evaluate the game using **positive** values, and opponent does so using **negative** values (zero-sum)

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Game strategies

- My gain is my opponent's loss (and vice versa)
- If the position value is 50 for me, it should be -50 for my opponent.
- If I reach $+\infty$, I win; if my opponent reaches $-\infty$, he/she wins.
- So I want to **MAXIMIZE** my score, while my opponent wants to **MINIMIZE** the score
- Thus, Minimax.

Introducing the players!

For the following slides, assume:

- It's blue's turn!
- **MIN** wants to minimize the value
- **MAX** wants to maximize the value

Minimax, Code


```
int minimax(Position p, boolean is_max) {
  if (p is a leaf) {
    // always position value of MAX
    return p.evaluate();
  }
  if (is_max) { // MAX
    int bestValue = −∞
    for (move in p.getMoves()) {
      p.applyMove();
      int value = minimax(p, is_max);
      p.undoMove();
      if (value > bestValue) {
        bestValue = value;
      }
    }
  } else { // MIN
    int bestValue = ∞
    for (move in p.getMoves()) {
      p.applyMove();
      int value = minimax(p, is_max);
      p.undoMove();
      if (value < bestValue) {
        bestValue = value;
      }
    }
  }
}
```

The highlighted parts are the only differences!

How do we simplify Minimax?

A fact

$$\max(a, b) = -\min(-a, -b)$$

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Change Minimax Code

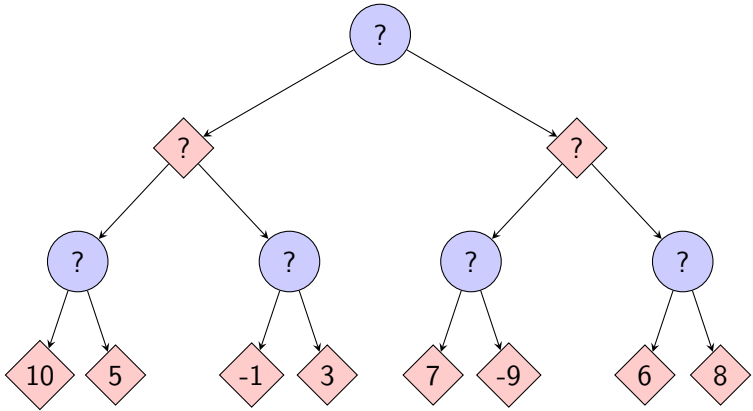
Then...

- For MAX player's turn, we negate the negative values returned by MIN, and find max
- For MIN player's turn, we negate the positive values returned by MAX, and find max, which is equivalent to find min.
- Now both players are maximizing, we can use the same piece of code.

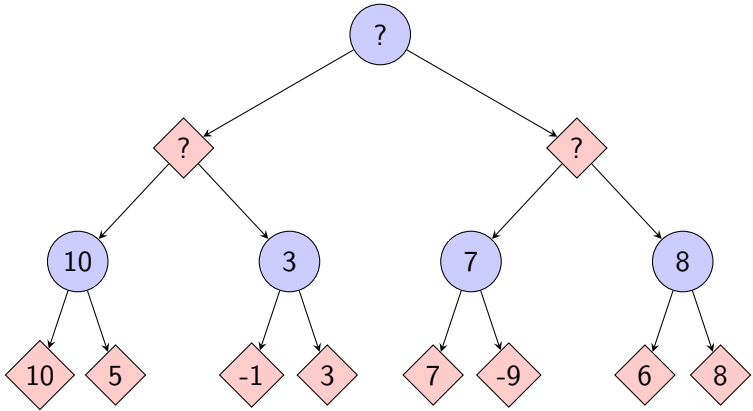
Code from your Game handout:

```
int minimax(Position p) {
  if (p is a leaf) {
    // position value of current player
    return p.evaluate();
  }
  int bestValue = −∞
  for (move in p.getMoves()) {
    p.applyMove();
    int value = -minimax(p);
    p.undoMove();
    if (value > bestValue) {
      bestValue = value;
    }
  }
}
```

Minimax Example



Minimax Example



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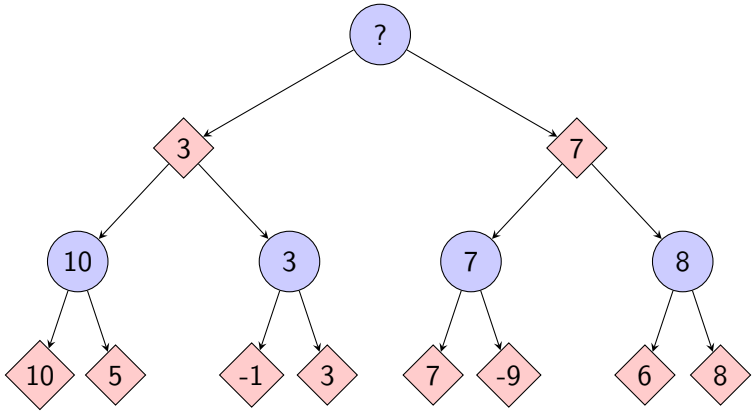
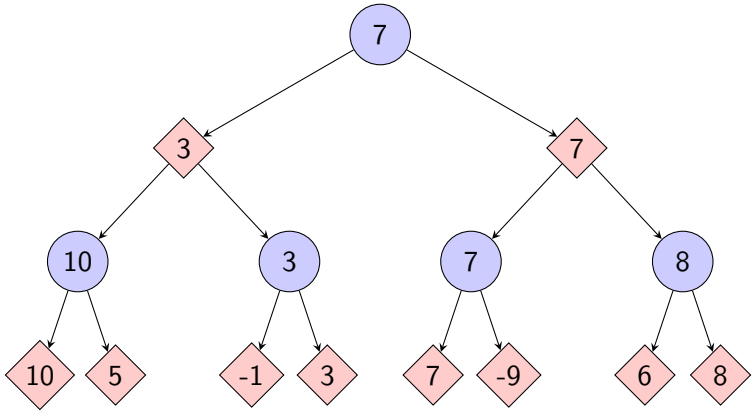
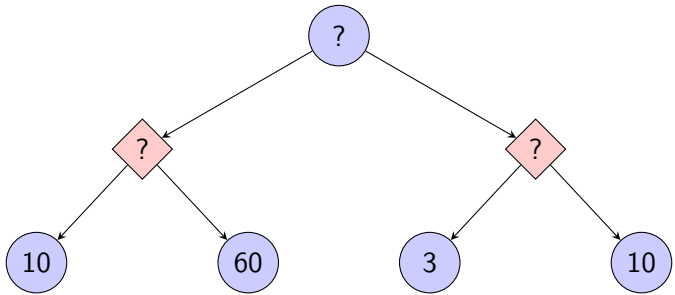


Minimax Example



Did we need to look at every leaf node?



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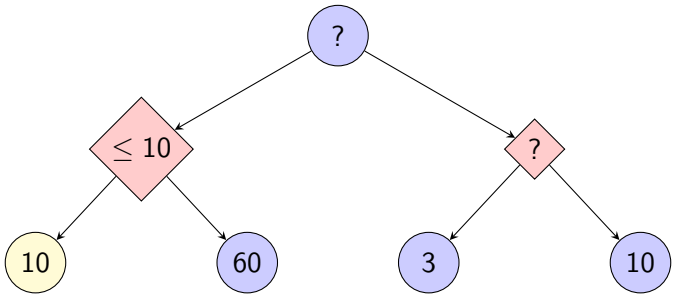

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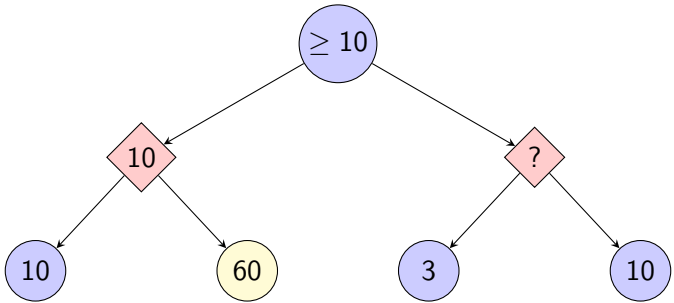

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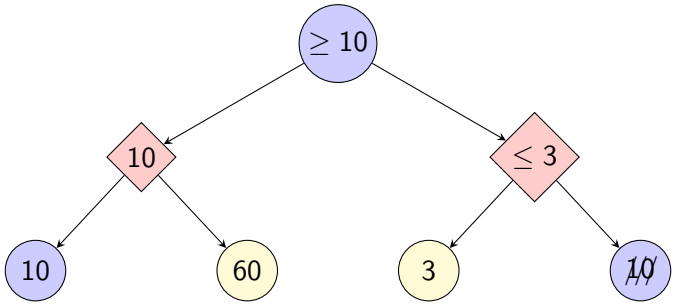


Now, without looking at 10, we know that the minimizer will give a score that is ≤ 3, yet the root maximizer already has a ≥ 10 guarantee. So we don't need to look at 10 really.

To formalize this process...

Alpha beta pruning

We are going to use two helper values:

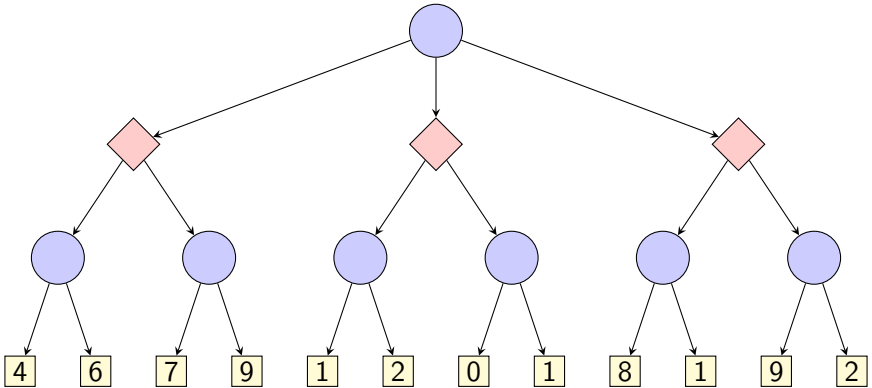
- α: best option **along the path to the root** for MAX
- β: best option **along the path to the root** for MIN

Pruning when...

- If the value of a MAX node is larger than β, or
- if the value of a MIN node is smaller than α

Overall, this means when α is larger than β, we prune the children of the current node.

Alpha beta demo[1]



[1] https://www.youtube.com/watch?v=xBXHtz4Gbdo&t=614s