Consider the following code:

```c
f(n) {
    if (n <= 1) {
        return 0;
    }
    int result = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            result += j;
        }
    }
    return f(n/2) + result + f(n/2);
}
```

(a) Find a recurrence for the time complexity of \( f(n) \).

**Solution:**

We look at the three separate cases (base case, non-recursive work, recursive work):

- The base case is \( \mathcal{O}(1) \), because we only do a return statement.
- The non-recursive work is \( \mathcal{O}(1) \) for the assignments and if tests and \( \sum_{i=0}^{n} i = \frac{n(n+1)}{2} \) for the for loops.
- The recursive work is \( 2T(n/2) \).

Putting these together, we get:

\[
T(n) = \begin{cases} 
1 & \text{if } n = 1 \\
2T(n/2) + \frac{n(n+1)}{2} & \text{otherwise}
\end{cases}
\]

(b) Find a Big-\( \mathcal{O} \) bound for your recurrence.

**Solution:**

The recursion tree has \( \lg(n) \) height, and each level of the tree does \( \left( \frac{n^2}{2^i} \right) \) work.

Note that the total work is then \( n^2 \sum_{i=0}^{\lg(n)} \left( \frac{1}{2^i} \right) = n^2 \sum_{i=0}^{\lg(n)} \left( \frac{1}{2^i} \right) < n^2 \sum_{i=0}^{\infty} \left( \frac{1}{2^i} \right) = \frac{n^2}{1 - \frac{1}{2}} \in \mathcal{O}(n^2) \).

So, \( T(n) \in \mathcal{O}(n^2) \).