0. Heaps

Insert 10, 7, 15, 17, 12, 20, 6, 32 into a min heap.
Now, insert the same values into a max heap.
Now, insert the same values into a min heap, but use Floyd’s buildHeap algorithm.
1. Big-Oh Proofs

For each of the following, prove that \( f \in O(g) \).

(a) \( f(n) = 7n \) \hspace{1cm} g(n) = \frac{n}{10}

(b) \( f(n) = 1000 \) \hspace{1cm} g(n) = 3n^3

(c) \( f(n) = 7n^2 + 3n \) \hspace{1cm} g(n) = n^4

(d) \( f(n) = n + 2n \log n \) \hspace{1cm} g(n) = n \log n
2. Is Your Program Running? Better Catch It!

For each of the following, determine the asymptotic worst-case runtime in terms of $n$.

(a)

```c
int x = 0;
for (int i = n; i >= 0; i--) {
    if ((i % 3) == 0) {
        break;
    } else {
        x += n;
    }
}
```

(b)

```c
int x = 0;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < (n * n / 3); j++) {
        x += j;
    }
}
```

(c)

```c
int x = 0;
for (int i = 0; i <= n; i++) {
    for (int j = 0; j < (i * i); j++) {
        x += j;
    }
}
```

3. Induction Shminduction

Prove $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$ by induction on $n$. 
4. The Implications of Asymptotics
For each of the following, determine if the statement is true or false.
(a) \( f(n) \in \Theta(g(n)) \rightarrow f(n) \in \mathcal{O}(g(n)) \)

(b) \( f(n) \in \Theta(g(n)) \rightarrow g(n) \in \Theta(f(n)) \)

(c) \( f(n) \in \Omega(g(n)) \rightarrow g(n) \in \mathcal{O}(f(n)) \)

5. Asymptotic Analysis
For each of the following, determine if \( f \in \mathcal{O}(g) \), \( f \in \Omega(g) \), \( f \in \Theta(g) \), several of these, or none of these.
(a) \( f(n) = \log n \) \hspace{1cm} g(n) = \log \log n 

(b) \( f(n) = 2^n \) \hspace{1cm} g(n) = 3^n 

(c) \( f(n) = 2^{2n} \) \hspace{1cm} g(n) = 2^n