CSE 332: Data Structures & Parallelism

P, NP, NP-Complete

(part 2)

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Today’s Agenda

- A Few Problems:
  - Euler Circuits
  - Hamiltonian Circuits
- Intractability: P and NP
- NP-Complete
- What now?

A Glimmer of Hope

- If given a candidate solution to a problem, we can **<u>check if that solution is correct in polynomial-time</u>**, then **maybe** a polynomial-time solution exists?

- Can we do this with Hamiltonian Circuit?
  - Given a candidate path, is it a Hamiltonian Circuit?

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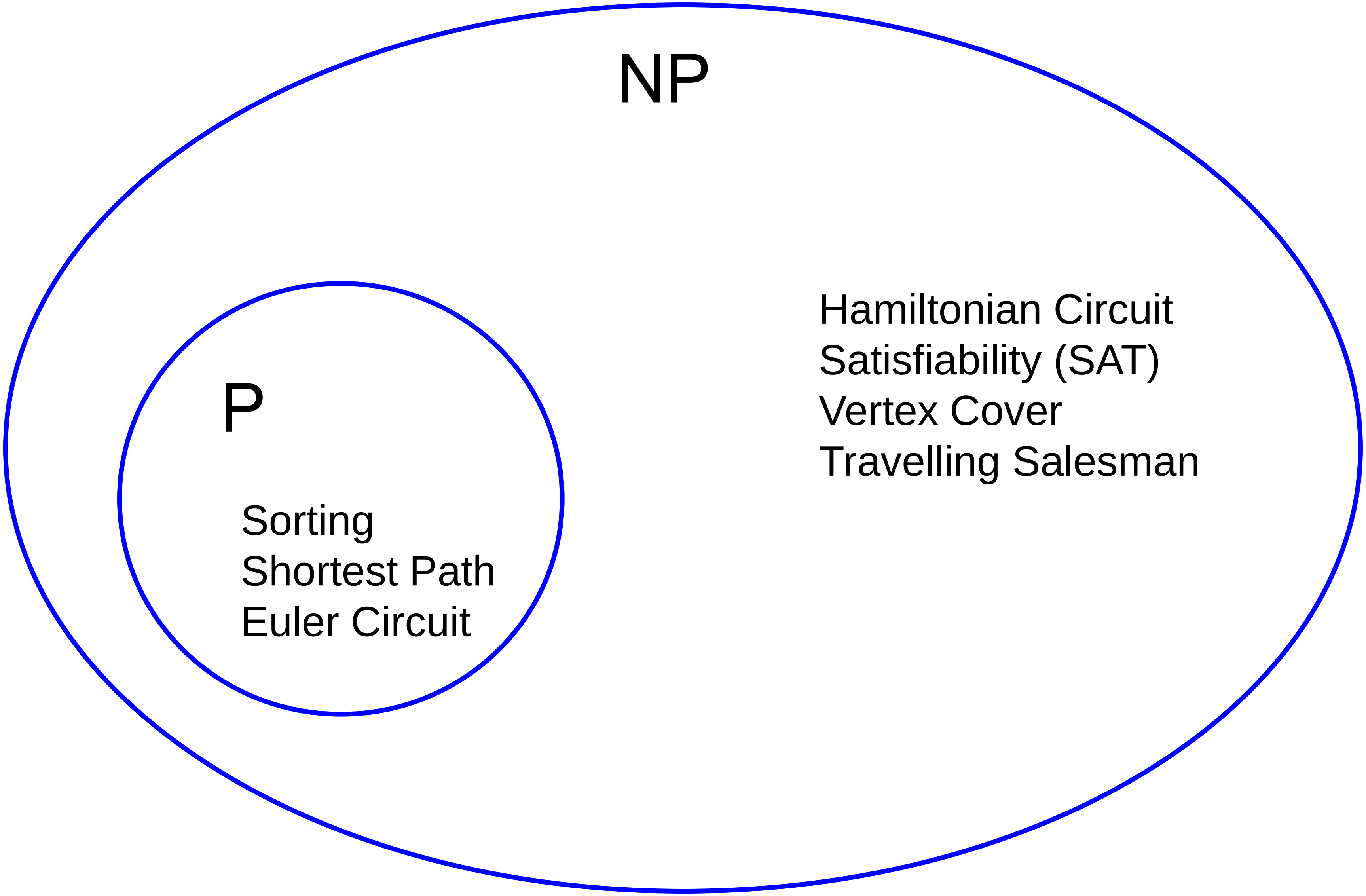
- Can we do this with Hamiltonian Circuit?
  - Given a candidate path, is it a Hamiltonian Circuit? just check if all vertices are visited exactly once in the candidate path

The Complexity Class NP

- *Definition*: NP is the set of all problems for which a given *candidate solution* can be *tested* in polynomial time

- Examples of problems in NP:
  - *Hamiltonian circuit:* Given a candidate path, can test in linear time if it is a Hamiltonian circuit
  - *Vertex Cover:* Given a subset of vertices, do they cover all edges?
  - *All problems that are in P (why?)*


NP
P
Sorting
Shortest Path
Euler Circuit
Hamiltonian Circuit
Satisfiability (SAT)
Vertex Cover
Travelling Salesman

Why do we call it “NP”?

- NP stands for *Nondeterministic Polynomial time*
  - Why “nondeterministic”? Corresponds to algorithms that can guess a solution (if it exists), the solution is then verified to be correct in polynomial time
  - Can also think of as allowing a special operation that allows the algorithm to magically guess the right choice at each branch point.
  - Nondeterministic algorithms don’t exist – purely theoretical idea invented to understand how hard a problem could be

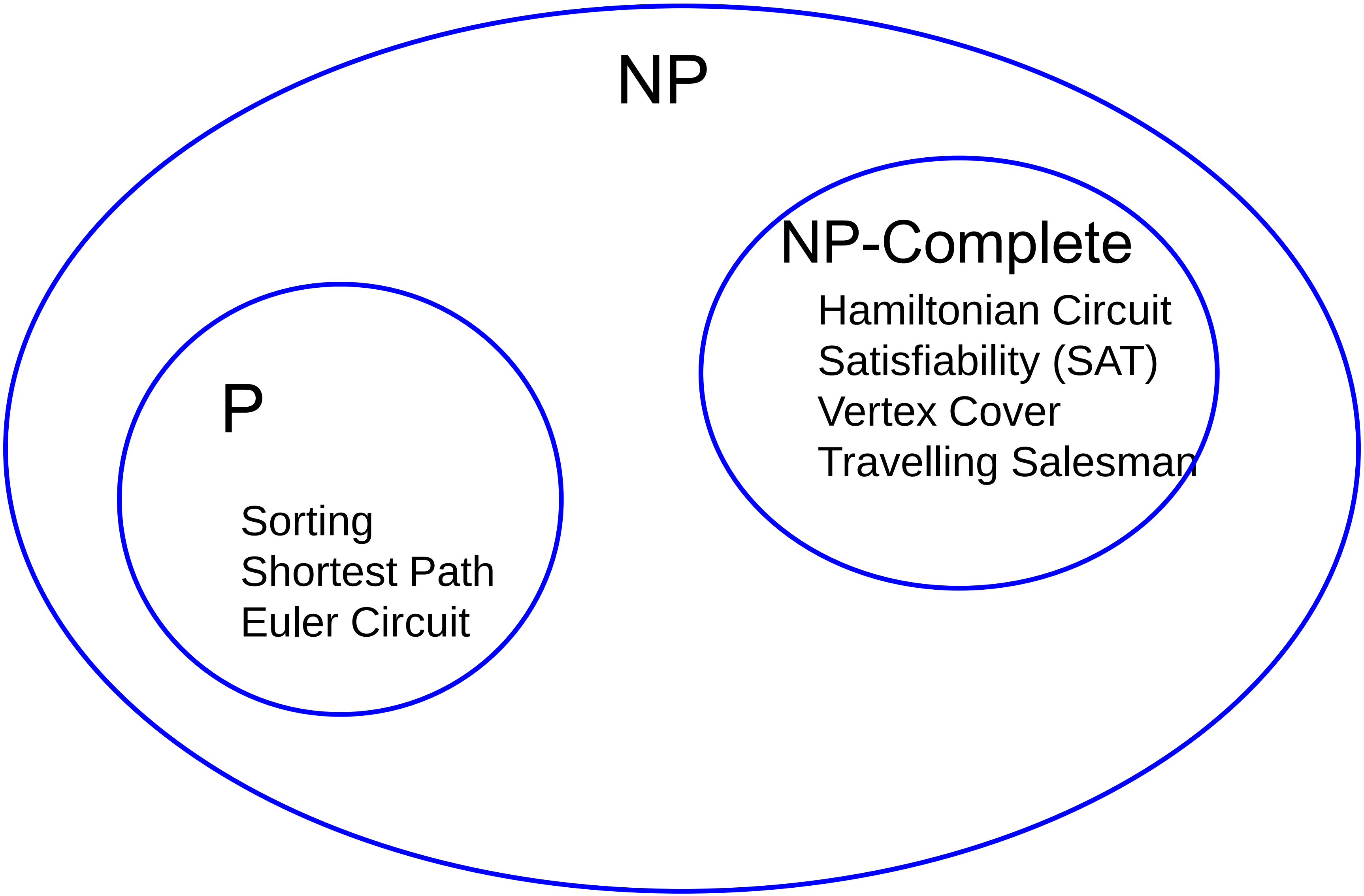
Your Chance to Win a Turing Award!

It is generally believed that P $\neq$ NP,
i.e. there are problems in NP that are **not** in P

- But no one has been able to show even one such problem!
- This is the fundamental open problem in theoretical computer science
- Nearly everyone has given up trying to prove it. Instead, theoreticians prove theorems about what follows once we assume P $\neq$ NP !

NP-completeness

- Set of problems in NP that (we are pretty sure) ***cannot*** be solved in polynomial time.
- These are thought of as the **hardest** problems in the class NP.
- **Interesting fact:** If any one NP-complete problem could be solved in polynomial time, then ***all*** NP-complete problems could be solved in polynomial time.
- Also: If any NP-complete problem is in P, then all of NP is in P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Saving Your Job

- Try as you might, every solution you come up with for the Hamiltonian Circuit problem runs in exponential time…..
- You have to report back to your boss.
- Your options:
  - Keep working
  - Come up with an alternative plan…

In general, what to do with a Hard Problem

- Your problem seems really hard.
- If you can transform a known NP-complete problem into the one you're trying to solve, then you can stop working on your problem!

Your Third Task

- Your boss buys your story that others couldn't solve the last problem.
- Again, your company has to send someone by car to a set of cities. There is a road between every pair of cities.
- The primary cost is distance traveled (which translates to fuel costs).
- Your boss wants you to figure out <u>*how to drive to each city exactly once*</u>, then return to the first city, while <u>*staying within a fixed mileage budget k*</u>.

Travelling Salesman Problem (TSP)

- Your third task is basically TSP:
  - Given <u>complete</u> weighted graph G, integer k.
  - Is there a cycle that visits all vertices with cost <= k?
- One of the canonical problems.

- Note difference from Hamiltonian cycle:
  - graph is complete
  - we care about weight.

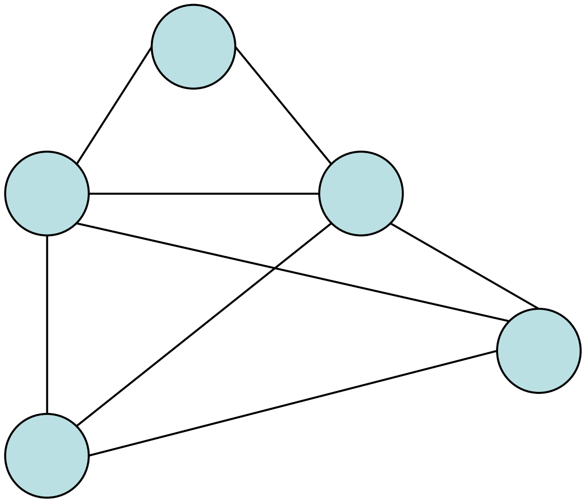
Transforming Hamiltonian Cycle to TSP

- We can "reduce" Hamiltonian Cycle to TSP.
- Given graph G=(V, E):
  - Assign weight of 1 to each edge
  - Augment the graph with edges until it is a complete graph G'=(V, E')
  - Assign weights of 2 to the new edges
  - Let k = |V|.

Notes:

- The transformation must take polynomial time
- You reduce the known NP-complete problem into your problem (not the other way around)
- In this case we are assuming Hamiltonian Cycle is our known NP-complete problem (in reality, both are known NP-complete)

Example



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Input to Hamiltonian Circuit Problem

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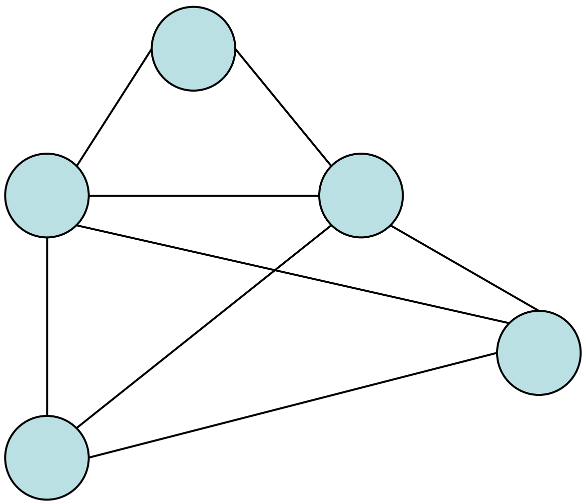

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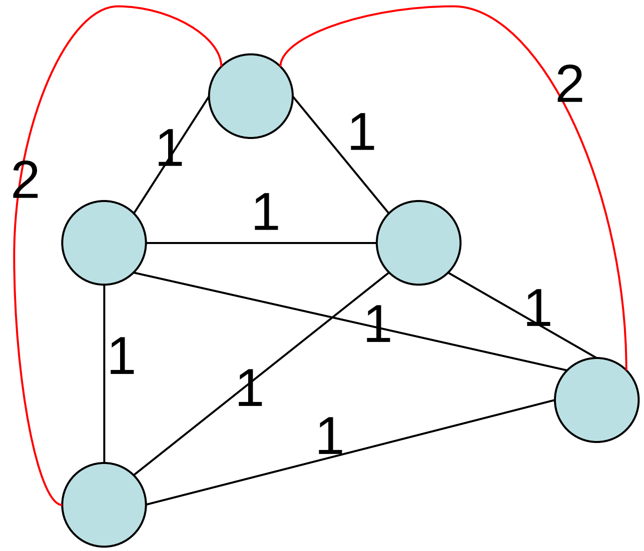
Polynomial time transformation



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Input to Hamiltonian Circuit Problem

Input to Traveling Salesman Problem

Polynomial-time transformation

- G' has a TSP tour of weight |V| iff
  G has a Hamiltonian Cycle.
- What was the cost of transforming HC into TSP?

- In the end, because there is a polynomial time transformation from HC to TSP, we say *TSP is "at least as hard as" Hamiltonian cycle.*

What do we do about it?

- Approximation Algorithm:
  - Can we get an efficient algorithm that guarantees something *close* to optimal? (e.g. Answer is guaranteed to be within 1.5x of Optimal, but solved in polynomial time).
- Restrictions:
  - Many hard problems are easy for restricted inputs (e.g. graph is always a tree, degree of vertices is always 3 or less).
- Heuristics:
  - Can we get something that seems to work well (good approximation/fast enough) *most* of the time? (e.g. In practice, n is small-ish)

Great Quick Reference

- *Computers and Intractability: A Guide to the Theory of NP-Completeness*, by Michael S. Garey and David S. Johnson