CSE332: Data Abstractions
Final Review

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Adapted from slides by Hye In Kim

Final Logistics

- Final on Wednesday, March 18th
  - Time: 12:30-2:20pm in Kane 220
    - No notes or no books
  - Info on website under “Final Exam”

Topics (short list)

- Sorting
- Graphs
- Parallelization
- Concurrency
- Amortized Analysis
- P, NP, NP-completeness
- Material in Midterm
  (fair game but not the focus)

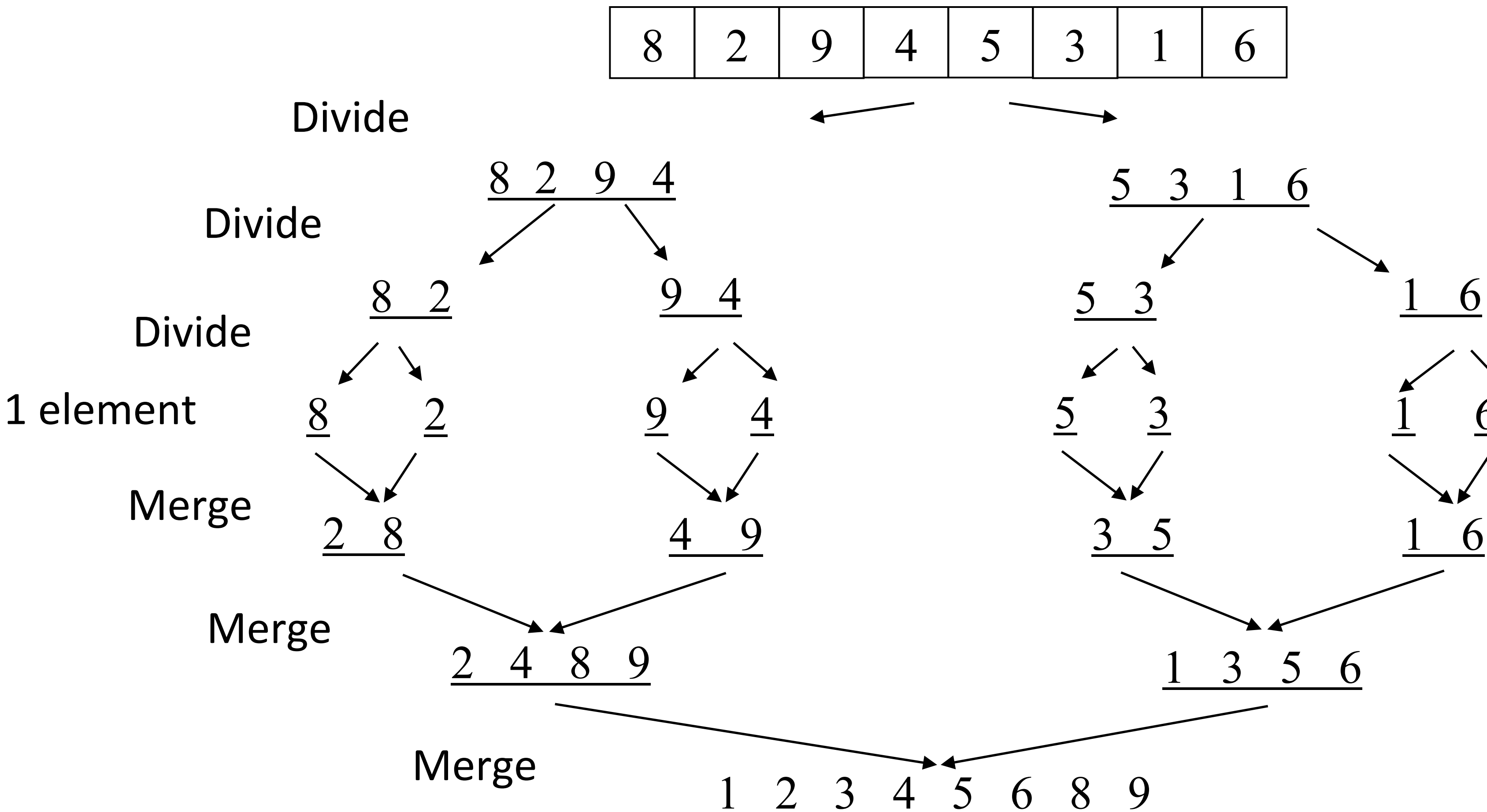
Preparing for the Exam

- Written Homework a good indication of what could be on exam
- Check out previous quarters' exams
  - 332 exams
  - 326 exams differ quite a bit
  - Final info site has links
- Make sure you:
  - Understand the key concepts
  - Can perform the key algorithms

Sorting Topics

- Know
  - Insertion & Selection sorts - $O(n^2)$
  - Heap Sort - O(n log n)
  - Merge Sort - O(n log n)
  - Quick Sort - O(n log n) on average
  - Bucket Sort & Radix Sort
- Know run-times
- Know how to carry out the sort
- Lower Bound for Comparison Sort
  - Cannot do better than O(n log n)
  - Won't be ask to give full proof
  - But may be asked to use similar techniques
  - Be familiar with the ideas

Mergesort example: Merge as we return from recursive calls



We need another array in which to do each merging step.
Merge results into there, then copy back to original array.

Graph Topics

- Graph Basics
  - Definition, weights, directedness, degree
  - Paths, cycles
  - Connectedness (directed vs undirected)
  - ‘Tree’ in a graph sense
  - DAGs
- Graph Representations
  - Adjacency List
  - Adjacency Matrix
  - What each is, how to use it
- Graph Traversals
  - Breadth-First
  - Depth-First
  - What data structures are associated with each?

Graph Topics

- Topological Sort
- Dijkstra’s Algorithm
  - Doesn’t play nice with negative weights
- Minimum Spanning Trees
  - Prim’s Algorithm
  - Kruskal’s Algorithm
- Know algorithms
- Know run-times

Dijkstra's Algorithm Overview

Given a weighted graph and a vertex in the graph (call it A), find the shortest path from A to each other vertex

- Cost of path defined as sum of weights of edges
- Negative edges not allowed

vertex	known?	cost	path
A		0	
B		??	
C		??	
D		??	

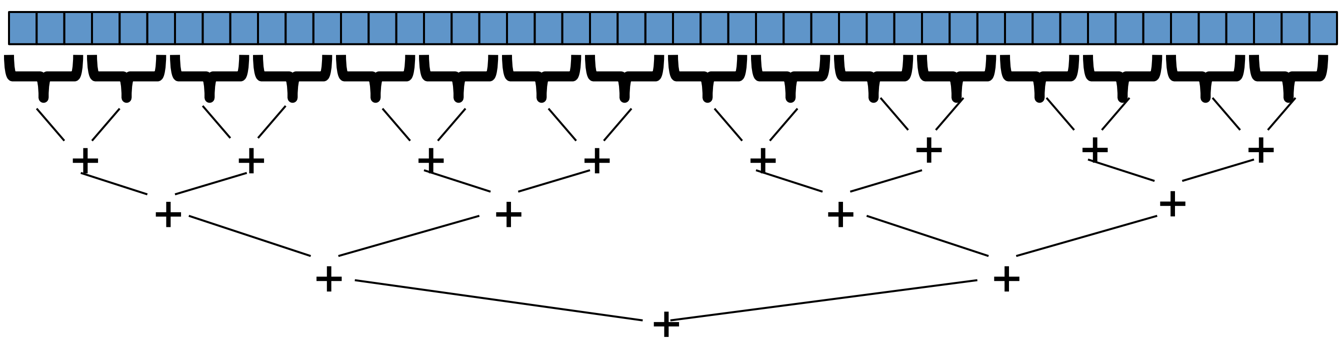
- The algorithm:
  - Create a table like this:
  - Init A's cost to 0, others infinity (or just '??')
  - While there are unknown vertices:
    - Select unknown vertex w/ lowest cost (A initially)
    - Mark it as known
    - Update cost and path to all unknown vertices adjacent to that vertex

Parallelism

- Fork-join parallelism
  - Know the concept; diff. from making lots of threads
  - Be able to write pseudo-code
  - Reduce: parallel sum, multiply, min, find, etc.
  - Map: bit vector, string length, etc.
- Work & span definitions
- Speed-up & parallelism definitions
- Justification for run-time, given tree
- Justification for ‘halving’ each step
- Amdahl’s Law
- Parallel Prefix
  - Technique
  - Span
  - Uses: Parallel prefix sum, filter, etc.
- Parallel Sorting

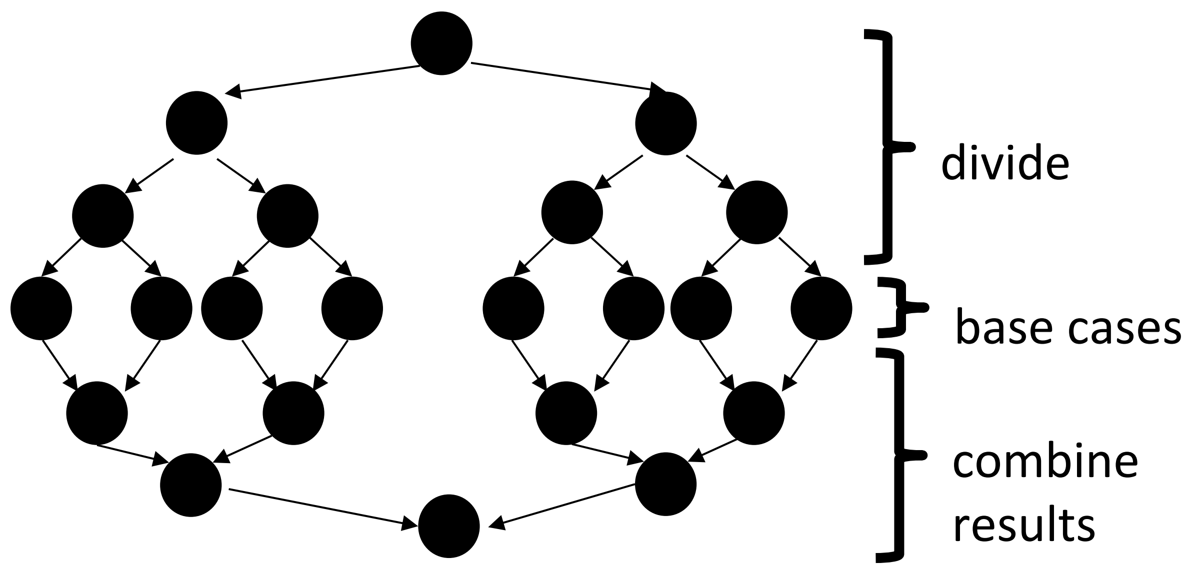
Parallelism Overview

- We say it takes time $\mathbf{T_P}$ to complete a task with P processors
- Adding together an array of n elements would take O(n) time, when done sequentially (that is, P=1)
  - Called the **work**; $\mathbf{T_1}$
- If we have 'enough' processors, we can do it much faster; O(logn) time
  - Called the **span**; $\mathbf{T_\infty}$



Considering Parallel Run-time

Our **`fork`** and **`join`** frequently look like this:



- Each node takes O(1) time
  - Even the base cases, as they are at the cut-off
- Sequentially, we can do this in O(n) time; O(1) for each node, ~3n nodes, if there were no cut-off (linear # on base case row, halved each row up/down)
- Carrying this out in (perfect) parallel will take the time of the longest branch; ~2logn, if we halve each time

Some Parallelism Definitions

- ***Speed-up*** on **P** processors: $\mathbf{T_1 / T_P}$
- We often assume perfect linear speed-up
  - That is, $\mathbf{T_1 / T_P} = P$; w/ 2x processors, it's twice as fast
  - 'Perfect linear speed-up 'usually our goal; hard to get in practice
- ***Parallelism*** is the maximum possible speed-up: $\mathbf{T_1 / T_\infty}$
  - At some point, adding processors won't help
  - What that point is depends on the span

The ForkJoin Framework Expected Performance

If you write your program well, you can get the following expected performance:

$$\mathbf{T_P \leq (T_1 / P) + O(T_\infty)}$$

– T_1/P for the overall work split between P processors
  - P=4? Each processor takes 1/4 of the total work
– $O(T_\infty)$ for merging results
  - Even if P=∞, then we still need to do $O(T_\infty)$ to merge results

- *What does it mean??*

– We can get decent benefit for adding more processors; effectively linear speed-up at first (expected)
– With a large # of processors, we're still bounded by $\mathbf{T_\infty}$; that term becomes dominant

Amdahl's Law

Let the ***work*** (time to run on 1 processor) be 1 unit time

Let **S** be the portion of the execution that **cannot** be parallelized

Then: $\mathbf{T_1 = S + (1\text{-}S) = 1}$

Then: $\mathbf{T_P = S + (1\text{-}S)/P}$

Amdahl's Law: The overall ***speedup*** with **P** processors is:

$$\mathbf{T_1 / T_P = 1 / (S + (1\text{-}S)/P)}$$

And the ***parallelism*** (infinite processors) is:

$$\mathbf{T_1 / T_\infty = 1 / S}$$

Parallel Prefix Sum

- Given an array of numbers, compute an array of their running sums in *O(logn)* span
- Requires 2 passes (each a parallel traversal)
  - First is to gather information
  - Second figures out output

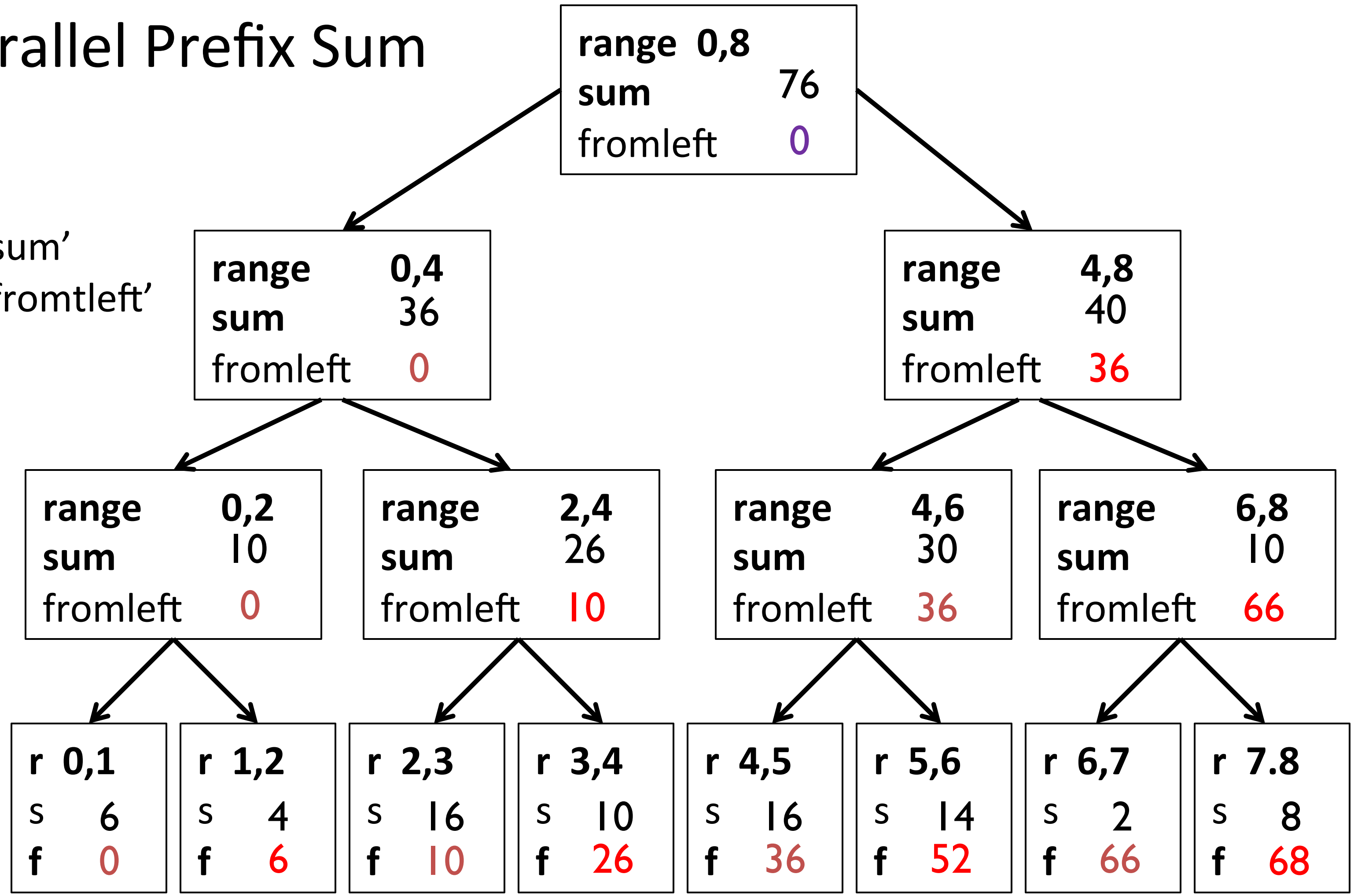
`input`	6	4	16	10	16	14	2	8
`output`	6	10	26	36	52	66	68	76

Parallel Prefix Sum

Two passes:

1) Compute ‘sum’

2) Compute ‘fromtleft’



input	6	4	16	10	16	14	2	8
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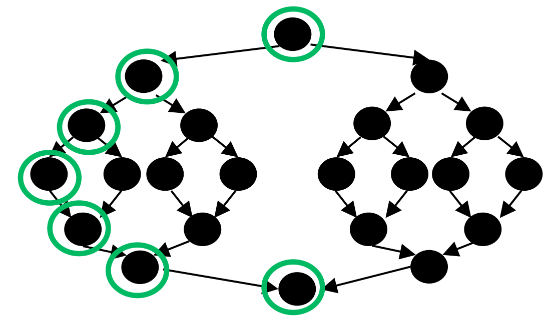
Parallel Quicksort



2 optimizations:

1. Do the two recursive calls in parallel
   - Now recurrence takes the form:

     $O(n) + 1T(n/2)$

     So $O(n)$ span
2. Parallelize the partitioning step
   - Partitioning normally $O(n)$ time
   - Recall that we can use Parallel Prefix Sum to 'filter' with $O(logn)$ span
   - Partitioning can be done with 2 filters, so $O(logn)$ span for each partitioning step

These two parallel optimizations bring parallel quicksort to a span of $O(\log^2 n)$

Concurrency

- Race conditions
- Data races
- Synchronizing your code
  - Locks, Reentrant locks
  - Java's 'synchronize' statement
  - Readers/writer locks
  - Deadlock
  - Issues of critical section size
  - Issues of lock scheme granularity – coarse vs fine
- Knowledge of bad interleavings
- Be able to write pseudo-code for Java threads and, locks

Race Conditions

A race condition occurs when the computation result depends on scheduling (how threads are interleaved)

- If T1 and T2 happened to get scheduled in a certain way, things go wrong
- We, as programmers, cannot control scheduling of threads; result is that we need to write programs that work independent of scheduling

Race conditions are bugs that exist only due to concurrency

- No interleaved scheduling with 1 thread

Typically, problem is that some *intermediate state* can be seen by another thread; screws up other thread

- Consider a 'partial' insert in a linked list; say, a new node has been added to the end, but 'back' and 'count' haven't been updated

Data Races

- A ***data race*** is a specific type of ***race condition*** that can happen in 2 ways:
  - Two different threads can ***potentially*** write a variable at the same time
  - One thread can ***potentially*** write a variable while another reads the variable
  - Simultaneous reads are fine; not a data race, and nothing bad would happen
  - 'Potentially' is important; we say the code itself has a data race – it is independent of an actual execution
- Data races are bad, but we can still have a race condition, and bad behavior, when no data races are present

Readers/writer locks

0 ≤ writers ≤ 1 &&
0 ≤ readers &&
writers*readers==0

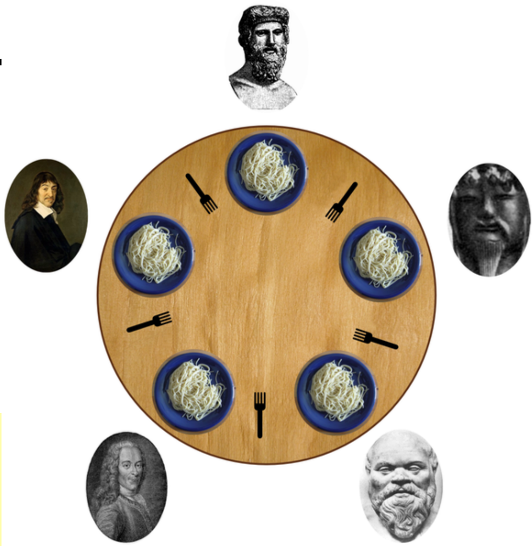
A new synchronization ADT: The **readers/writer lock**

- Idea: Allow any number of readers OR one writer
- This allows more concurrent access (multiple readers)
- A lock's states fall into three categories:
  - "not held"
  - "held for writing" by one thread
  - "held for reading" by *one or more* threads

- **`new`**: make a new lock, initially "not held"
- **`acquire_write`**: block if currently "held for reading" or "held for writing", else make "held for writing"
- **`release_write`**: make "not held"
- **`acquire_read`**: block if currently "held for writing", else make/keep "held for reading" and increment *readers count*
- **`release_read`**: decrement readers count, if 0, make "not held"

Deadlock

- As illustrated by the 'The Dining Philosophers' problem
- A deadlock occurs when there are threads **T1**
  - Each is waiting for a lock held by the next
  - **Tn** is waiting for a resource held by **T1**
- In other words, there is a cycle of waiting



```
class BankAccount {
  …
  synchronized void withdraw(int amt) {…}
  synchronized void deposit(int amt) {…}
  synchronized void transferTo(int amt,BankAccount a){
    this.withdraw(amt);
    a.deposit(amt);
  }
}
```

Consider simultaneous transfers from account x to account y, and y to x

Amortized Analysis

- To have an Amortized Bound of O(f(n)):
  - *There does not exist a series of M operations with run-time worse than O(M*f(n))*
- Amortized vs average case
- To prove: prove that no series of operations can do worse than O(M*f(n))
- To disprove: find a series of operations that's worse

P, NP, NP Completeness

- P: set of all problems that can be solved in polynomial time
  - sorting, shortest path, Euler circuit, etc.
- NP: set of all problems for which a given candidate solution can be tested in polynomial time
  - Hamiltonian Circuit, Vertex Cover, etc.

P=NP ???

- Currently no proof.
- It is generally believed that P≠NP
- Prove it for fame, fortune and a Turing Award!

NP-Complete

- Set of problems in NP that (we are pretty sure) cannot be solved in polynomial time.
- These are thought of as the hardest problems in the class NP.
- Interesting fact: If any one NP-Complete problem could be solved in polynomial time, then all NP-Complete problems could be solved in polynomial time.
- Even more: If any NP-Complete problem is in P, then all of NP is in P

Is my problem in P or NP?

- Reduce a known NP-Complete problem into your problem
- (not the other way around) via a transformation
  - The transformation must take polynomial time
- Now you can say your problem is at least as hard as a known NP-Complete problem

Working with NP Problems

- Approximation Algorithm
  - Can we get an efficient algorithm that guarantees something close to optimal? (e.g. Answer is guaranteed to be within 1.5x of Optimal, but solved in polynomial time).
- Restrictions
  - Many hard problems are easy for restricted inputs (e.g. graph is always a tree, degree of vertices is always 3 or less).
- Heuristics
  - Can we get something that seems to work well (good approximation/fast enough) most of the time? (e.g. In practice, n is small-ish)