CSE332: Data Abstractions

Section 5

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Section Agenda

- **Sorting Algorithms**
  - Insertion & Selection Sort
  - Merge & Quick Sort
  - Bucket & Radix Sort

- **Midterm practice problems**

Sorting Algorithms

Comparison & Non-comparison
based sorting

Sorting

- **Sorting**

  Rearranging elements in collection into a specific order

  - Can be solved in many ways

- **Comparison-based sorting**

  Determining order by comparing pairs of elements

  Insertion sort, selection sort, quick sort ...

Insertion Sort

Insertion Sort

- At kth step, insert kth element in correct position among the first k elements
- At kth step, the first k elements are sorted
- Works well when input is mostly sorted

Insertion sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

Insert 18

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

Insert 12

index	0	1	2	3	4	5	6	7
value	18	22	12	-4	58	7	31	42

Insert -4

index	0	1	2	3	4	5	6	7
value	12	18	22	-4	58	7	31	42

Insert 58

index	0	1	2	3	4	5	6	7
value	-4	12	18	22	58	7	31	42

Insertion sort example

Insert 7

index	0	1	2	3	4	5	6	7
value	-4	12	18	22	58	7	31	42

Insert 31

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	58	31	42

Insert 42

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	58	42

Sorted!

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	42	58

Insertion Sort Runtime

- **Base Case:** $T(1) = c$

  - Sorting 1 element take constant time

- **Recurrence Relation**

  - When input is sorted:

    Time for Sorting n elements

    = (Time for sorting n – 1 elements) + (1 comparisons)

$$T(n) = T(n-1) + 1, \qquad T(n) \in O(n)$$

Insertion Sort Runtime

- **Recurrence Relation**
  - When input is unsorted:

Time for Sorting n elements

= (Time for sorting n – 1 elements) + (n – 1 comparisons)

$T(n) = T(n\text{-}1) + (n\text{-}1)$, $T(n) \in O(n^2)$

Selection Sort

Selection Sort

- At kth step, find smallest value from unsorted items and place it in position k
- At kth step, the first k elements are sorted, and are the smallest elements

Selection sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

Min: -4

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

Min: 7

index	0	1	2	3	4	5	6	7
value	-4	18	12	22	58	7	31	42

Min: 12

index	0	1	2	3	4	5	6	7
value	-4	7	12	22	58	18	31	42

Min: 18

index	0	1	2	3	4	5	6	7
value	-4	7	12	22	58	18	31	42

Selection sort example

Min: 22

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	58	22	31	42

Min: 31

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	58	31	42

Min: 42

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	58	42

Sorted!

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	42	58

Selection Sort Runtime

- **Base Case:** $T(1) = c$

  - Sorting 1 element take constant time

- **Recurrence Relation**

  - At each step, work decrease by 1
  - At each step, need to compare all remaining elements to find the minimum

  $T(n) = T(n-1) + n,$ $T(n) \in O(n^2)$

Merge Sort

Merge Sort

- **Divide & Conquer**

- Divide into two roughly equal halves.

- Sort each halves

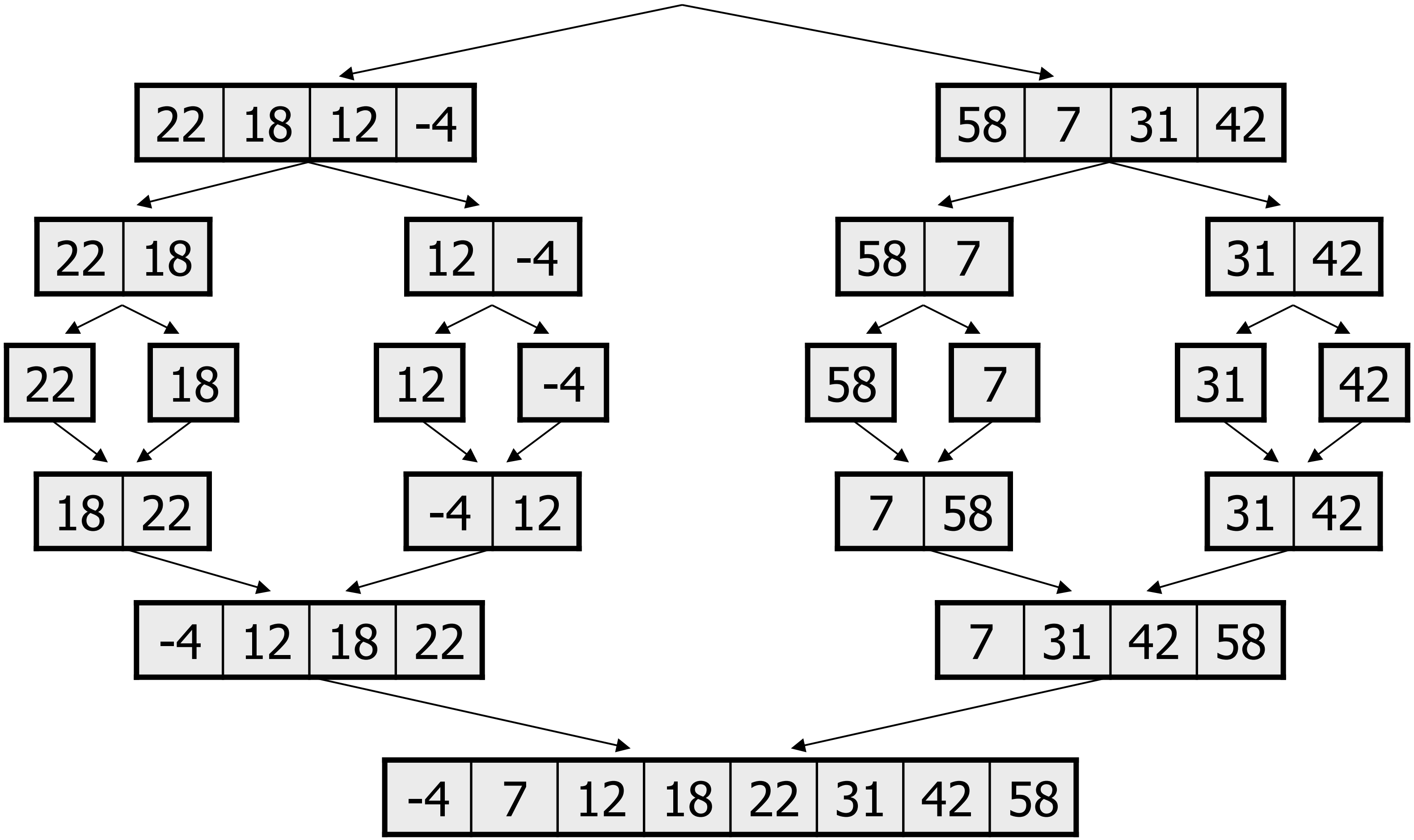
- Merge two sorted halves

- **Parallelizes Well**

- Multiple processors can work on different parts of array

Merge sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



22 | 18 | 12 | -4   58 | 7 | 31 | 42

22 | 18   12 | -4   58 | 7   31 | 42

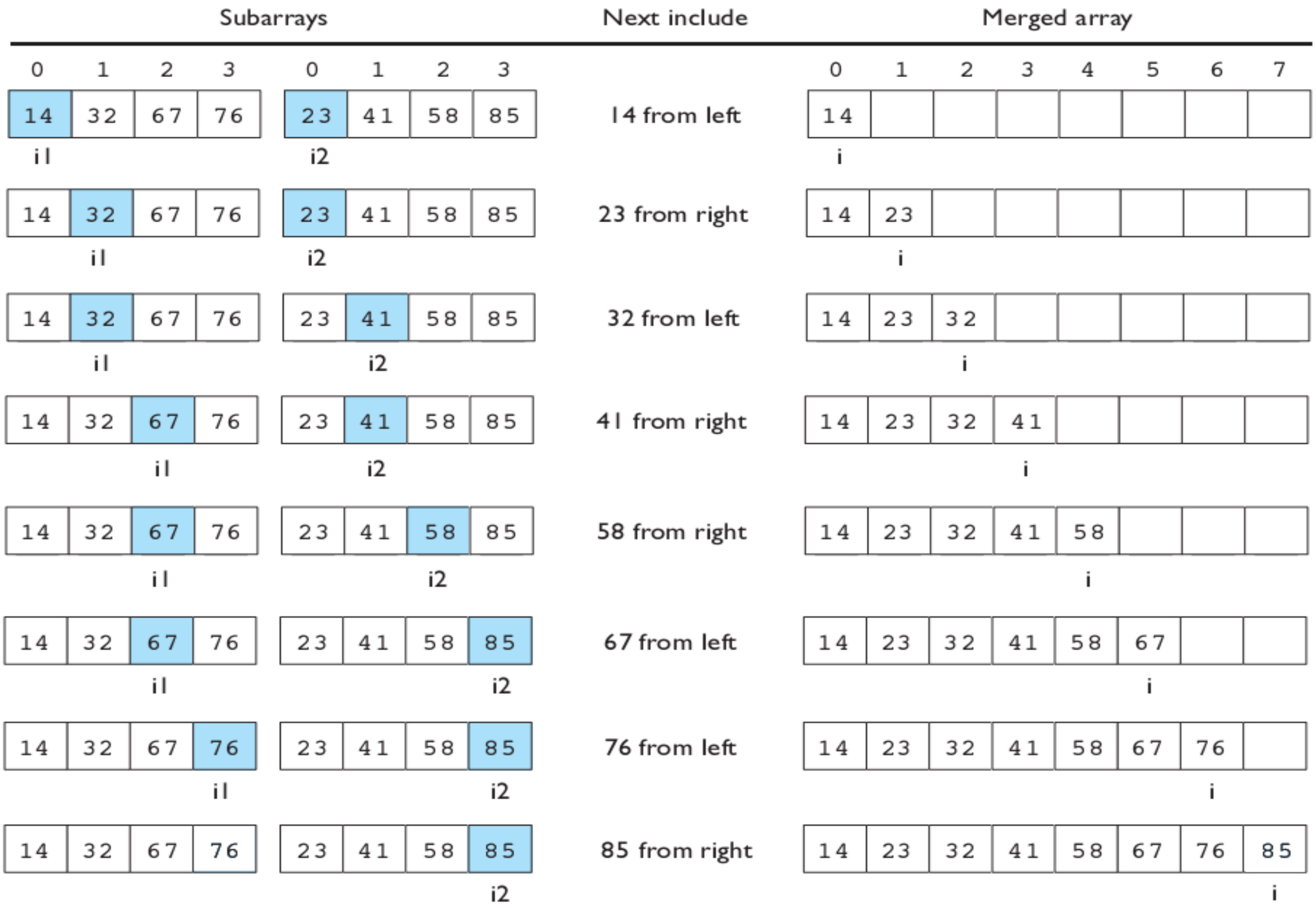
22   18   12   -4   58   7   31   42

18 | 22   -4 | 12   7 | 58   31 | 42

-4 | 12 | 18 | 22   7 | 31 | 42 | 58

-4 | 7 | 12 | 18 | 22 | 31 | 42 | 58

Merging sorted halves



Subarrays								Next include	Merged array							
0	1	2	3	0	1	2	3		0	1	2	3	4	5	6	7
14	32	67	76	23	41	58	85	14 from left	14							
i1				i2					i							
14	32	67	76	23	41	58	85	23 from right	14	23						
	i1			i2						i						
14	32	67	76	23	41	58	85	32 from left	14	23	32					
	i1				i2						i					
14	32	67	76	23	41	58	85	41 from right	14	23	32	41				
		i1			i2							i				
14	32	67	76	23	41	58	85	58 from right	14	23	32	41	58			
		i1				i2							i			
14	32	67	76	23	41	58	85	67 from left	14	23	32	41	58	67		
		i1					i2							i		
14	32	67	76	23	41	58	85	76 from left	14	23	32	41	58	67	76	
			i1				i2								i	
14	32	67	76	23	41	58	85	85 from right	14	23	32	41	58	67	76	85
							i2									i

Merge Sort Runtime

- **Base Case:** $T(1) = c$

  \- Sorting 1 element take constant time

- **Recurrence Relation**

  \- At each step, branch into two

  \- At each step, work decrease by half

  \- At each step, need to visit all elements

$T(n) = 2*T(n/2) + n,$ $T(n) \in O(n \log n)$

Quick Sort

Quick Sort

- **Divide & Conquer**

- Divide into two pieces

- Sort each piece

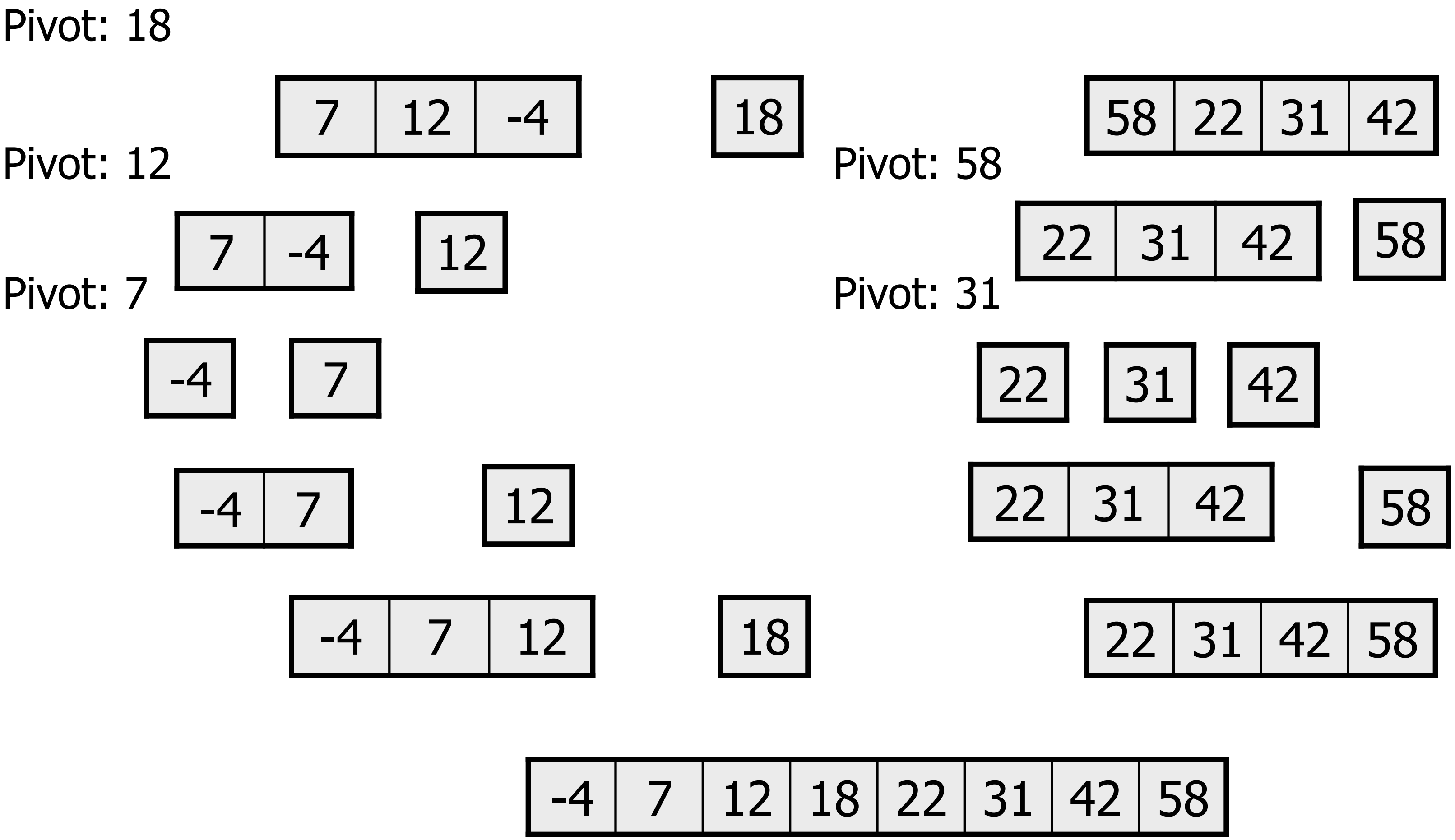
- Merge two sorted piece

- Pick pivot, partition into < pivot & > pivot

- Less copying & more comparisons compared to merge sort

Quick sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



Pivot: 18

7 12 -4 | 18 | 58 22 31 42

Pivot: 12

Pivot: 58

7 -4 | 12

22 31 42 | 58

Pivot: 7

Pivot: 31

-4 | 7

22 | 31 | 42

-4 7 | 12

22 31 42 | 58

-4 7 12 | 18 | 22 31 42 58

-4 7 12 18 22 31 42 58

Quick Sort Runtime

- **Base Case:** T(1) = c

  - Sorting 1 element take constant time

- **Recurrence Relation**

  - When pivot is the best:

    At each step, work decrease by half

    At each step, need to visit all elements

T(n) = 2*T(n/2) + n, T(n) $\in$ O(n log n)

Quick Sort Runtime

- **Recurrence Relation**
  - When pivot is the worst:

At each step, work decrease by 1

At each step, need to visit all elements

$T(n) = T(n-1) + n,$ $T(n) \in O(n^2)$

Bucket Sort

Bucket Sort

- **No Comparisons**

- Create a bucket for every possible elements in input
- Store counts for occurrence in corresponding bucket

- **Runtime:** O(n + k)

- k = range of possible values (size of bucket)
- Good for small k
- When k >>> n, space can be wasted

Bucket sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

First pass, find range (K): Min = -4, Max = 58

K = Max – Min + 1 = 63

Bucket	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bucket	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bucket	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bucket	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

Bucket sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

Second pass, Count occurrences

Bucket	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
count	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Bucket	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
count	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
Bucket	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
count	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
Bucket	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	

Bucket sort example

Bucket	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
count	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Bucket	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
count	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0

Bucket	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
count	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0

Bucket	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Third pass, Print occurrences

Sorted!

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	42	58

Radix Sort

Radix Sort

- **No Comparisons**

- Bucket sort on 1 digit at a time

- After k passes, last k digits are sorted

- **Runtime:** O(d*(n + k))

- k = radix (number of buckets)
- d = max number of digit = $\log_k$ (Max element)

Radix sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42

First pass, Sort by 1’s digit:

Digit	0	1	2	3	4	5	6	7	8	9
values					-4					
Digit	0	1	2	3	4	5	6	7	8	9
values		31	22 12 42					7	18 58	

Radix sort example

Digit	0	1	2	3	4	5	6	7	8	9
values					-4					
Digit	0	1	2	3	4	5	6	7	8	9
values		31	22 12 42					7	18 58	

Second pass, Sort by 10’s digit:

Digit	0	1	2	3	4	5	6	7	8	9
values	-4									
Digit	0	1	2	3	4	5	6	7	8	9
values	7	12 18	22	31	42	58				

Radix sort example

Digit	0	1	2	3	4	5	6	7	8	9
values	-4									
Digit	0	1	2	3	4	5	6	7	8	9
values	7	12 18	22	31	42	58				

Write out the values:

Sorted!

index	0	1	2	3	4	5	6	7
value	-4	7	12	18	22	31	42	58