CSE332: Data Abstractions

Lecture 9: Hashing

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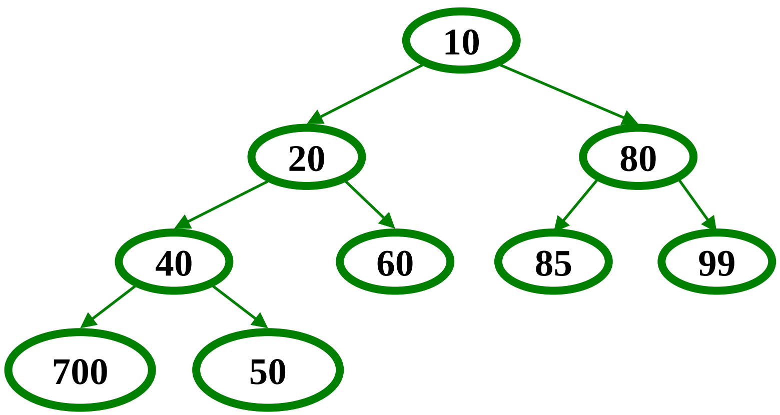
Winter 2012

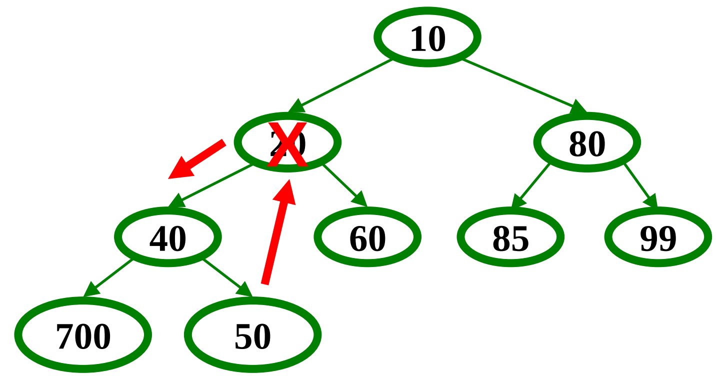
Administrative

- Midterm Review Poll
- Project 2a Due Wednesday
- Homework 4 Due Friday
- Feedback Plans

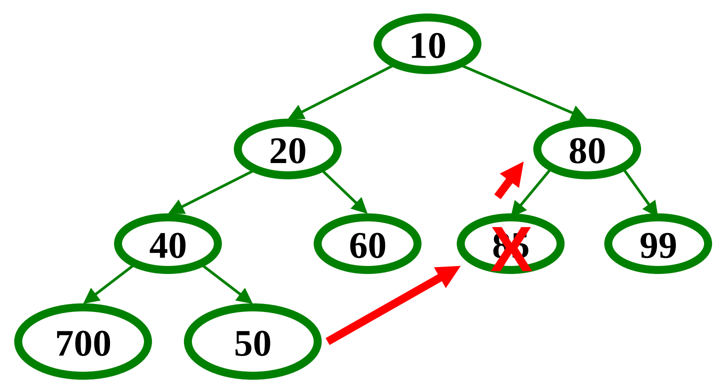
Homework 2, Problem 2



Need to percolate down



Also must percolate up



Open Addressing: Linear Probing

- Why not use up the empty space in the table?
- Store directly in the array cell (no linked list)
- How to deal with collisions?
- If **`h(key)`** is already full,
  - try **`(h(key) + 1) % TableSize`**. If full,
  - try **`(h(key) + 2) % TableSize`**. If full,
  - try **`(h(key) + 3) % TableSize`**. If full…
- Example: insert 38, 19, 8, 109, 10

0	/
1	/
2	/
3	/
4	/
5	/
6	/
7	/
8	/
9	/

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5	/
6	/
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9	/

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Open Addressing

This is *one example* of open addressing

In general, open addressing means resolving collisions by trying a sequence of other positions in the table

Trying the next spot is called probing

- We just did linear probing
  `h(key) + i) % TableSize`
- In general have some probe function **`f`** and use
  `h(key) + f(i) % TableSize`

Open addressing does poorly with high load factor λ

- So we want larger tables
- Too many probes means we lose our $O(1)$

Terminology

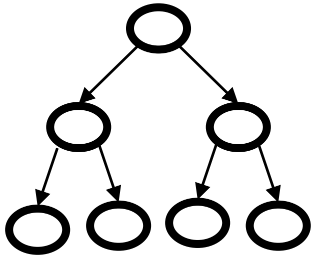
We and the book use the terms

- "chaining" or "separate chaining"
- "open addressing"

Very confusingly,

- "open hashing" is a synonym for "chaining"
- "closed hashing" is a synonym for "open addressing"

We also do trees upside-down



Other Operations

`insert` finds an open table position using a probe function

What about **`find`**?

- Must use same probe function to “retrace the trail” for the data
- Unsuccessful search when reach empty position

What about **`delete`**?

- ***Must*** use “lazy” deletion. Why?

- Marker indicates “no data here, but don’t stop probing”

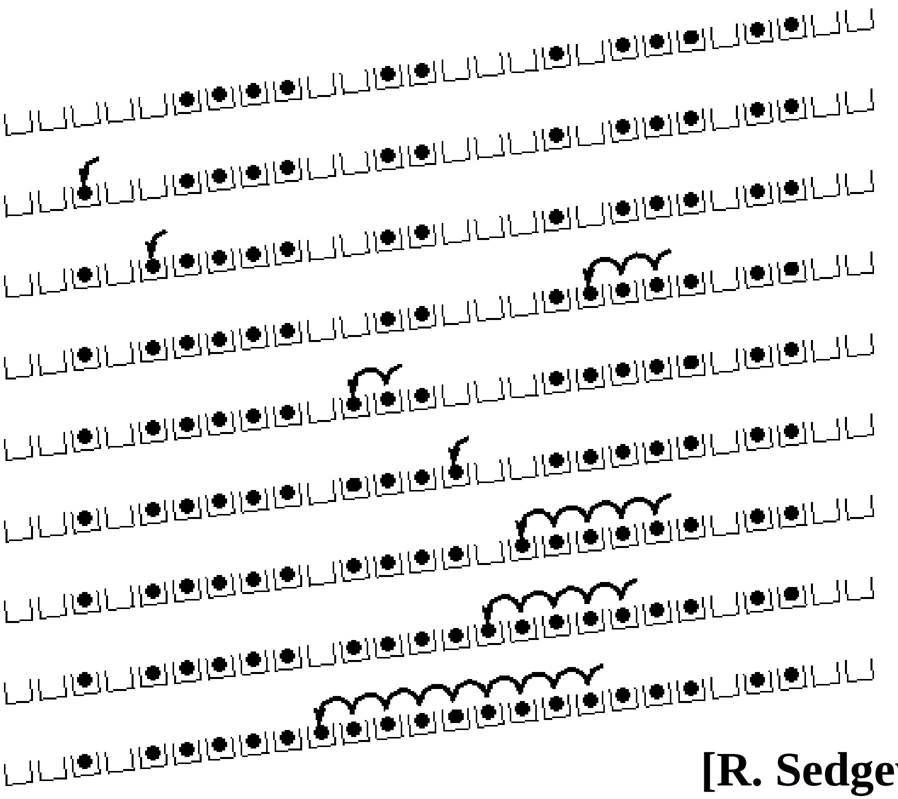
10	✘	/	**23**	/	/	**16**	✘	**26**

Primary Clustering

It turns out linear probing is a *bad idea*, even though the probe function is quick to compute (which is a good thing)

Tends to produce *clusters*, which lead to long probe sequences

- Called primary clustering
- Saw this starting in our example



[R. Sedgewick]

Analysis of Linear Probing

- Trivial fact: For any $\lambda < 1$, linear probing will find an empty slot
  - It is "safe" in this sense: no infinite loop unless table is full

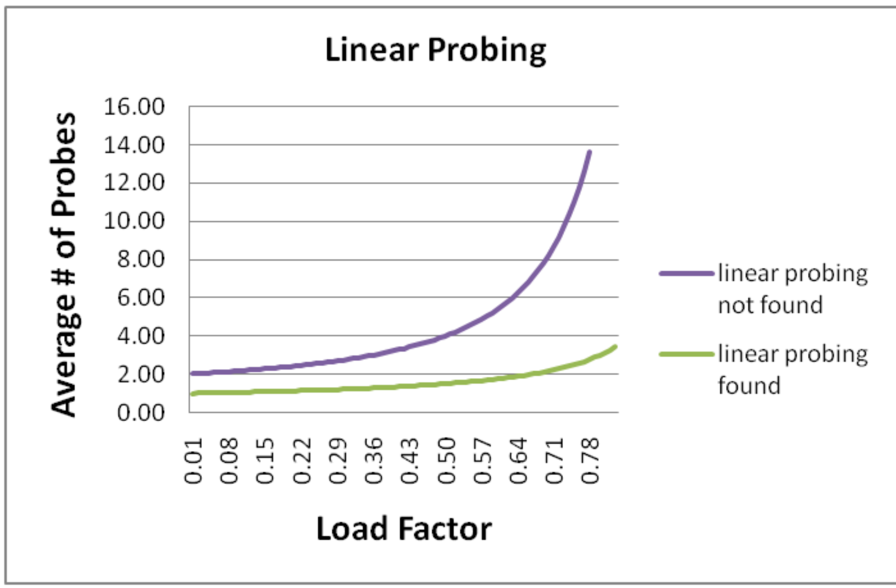
- Non-trivial facts we won't prove:

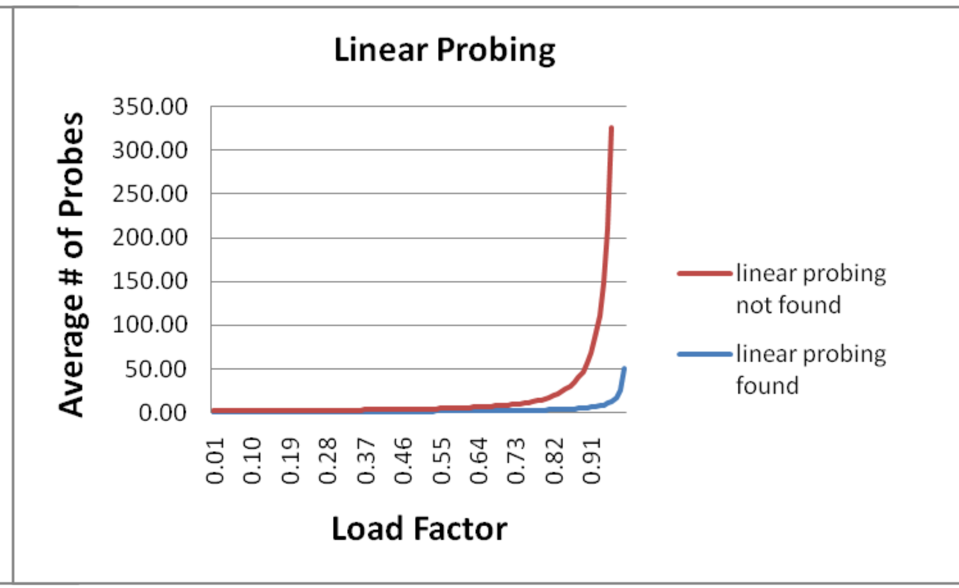
  Average # of probes given λ (in the limit as **`TableSize`** $\rightarrow \infty$)
  - Unsuccessful search: $$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$$
  - Successful search: $$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$$

- This is pretty bad: need to leave sufficient empty space in the table to get decent performance (let's look at a chart)

Analysis in Chart Form

- Linear-probing performance degrades rapidly as table gets full
  - Formula assumes “large table” but point remains





- Chaining performance was linear in λ and has no trouble with $\lambda > 1$

Open Addressing: Quadratic Probing

- We can avoid primary clustering by changing the probe function

  `(h(key) + f(i)) % TableSize`

  - For quadratic probing:

    `f(i) = i`^{2}

  - So probe sequence is:
    - 0th probe: `h(key) % TableSize`
    - 1st probe: `(h(key) + 1) % TableSize`
    - 2nd probe: `(h(key) + 4) % TableSize`
    - 3rd probe: `(h(key) + 9) % TableSize`
    - …
    - ith probe: `(h(key) + i`^{2}`) % TableSize`

- Intuition: Probes quickly “leave the neighborhood”

Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

TableSize=10

Insert:
89
18
49
58
79

Quadratic Probing Example

0	
1	
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Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	

TableSize = 7

Insert:

76	**(76 % 7 = 6)**
40	**(40 % 7 = 5)**
48	**(48 % 7 = 6)**
5	**(5 % 7 = 5)**
55	**(55 % 7 = 6)**
47	**(47 % 7 = 5)**

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Doh: For all *n*, **`(5 +(n*n)) % 7 is 0, 2, 5, or 6`**

Proof uses induction and $(n^2+5) \% 7 = ((n-7)^2+5) \% 7$

In fact, for all *c* and *k*, $(n^2+c) \% k = ((n-k)^2+c) \% k$

From Bad News to Good News

- After **`TableSize`** quadratic probes, we cycle through the same indices

- The good news:

  – For prime **`T`** and $0 \leq i,j \leq T/2$ where $i \neq j$,

  $$(h(key) + i^2) \% T \neq (h(key) + j^2) \% T$$

  – If **`T = TableSize`** is *prime* and λ < ½,
  quadratic probing will find an empty slot in at most **`T/2`** probes

  – If you keep λ < ½, no need to detect cycles

Clustering Reconsidered

- Quadratic probing does not suffer from primary clustering: quadratic nature quickly escapes the neighborhood
- But it’s no help if keys *initially hash to the same index*
  - Any 2 keys that hash to the same value will have the same series of moves after that
  - Called secondary clustering
- Can avoid secondary clustering with *a probe function that depends on the key*: double hashing

Open Addressing: Double Hashing

Idea: Given two good hash functions *h* and *g*, it is very unlikely that for some *key*, **`h(key) == g(key)`**

(**`h(key) + f(i)) % TableSize`**

- For double hashing:

  `f(i) = i*g(key)`

- So probe sequence is:
  - 0^{th} probe: **`h(key) % TableSize`**
  - 1^{st} probe: **`(h(key) + g(key)) % TableSize`**
  - 2^{nd} probe: **`(h(key) + 2*g(key)) % TableSize`**
  - 3^{rd} probe: **`(h(key) + 3*g(key)) % TableSize`**
  - …
  - i^{th} probe: **`(h(key) + i*g(key)) % TableSize`**

- Detail: Must make sure that **`g(key)`** cannot be **`0`**

Double Hashing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

T = 10 (TableSize)

Hash Functions:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

147

43

Double Hashing

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Doh:

3 + 0 = 3

3 + 5 = 8

3 + 10 = 13

3 + 15 = 18

3 + 20 = 23

3 + 25 = 28

Double Hashing Analysis

- Intuition:

  Because each probe is “jumping” by **`g(key)`** each time, we should both “leave the neighborhood” *and* “go different places from the same initial collision”

- But, as in quadratic probing, we could still have a problem where we are not “safe” (infinite loop despite room in table)

- It is known that this cannot happen in at least one case:
  - **`h(key) = key % p`**
  - **`g(key) = q – (key % q)`**
  - **`2 < q < p`**
  - **`p`** and **`q`** are prime

Where are we?

- Separate Chaining is easy
  - **`find`**, **`delete`** proportional to load factor on average
  - **`insert`** can be constant if just push on front of list

- Open addressing uses probing, has clustering issues as it gets full
  - Why use it:
    - Less memory allocation?
    - Run-time overhead for list nodes; array could be faster?
    - Easier data representation?

- Now:
  - Growing the table when it gets too full (aka “rehashing”)
  - Relation between hashing/comparing and connection to Java

Rehashing

- As with array-based stacks/queues/lists
  - If table gets too full, create a bigger table and copy everything
- With chaining, we get to decide what “too full” means
  - Keep load factor reasonable (e.g., < 1)?
  - Consider average or max size of non-empty chains?
- For open addressing, half-full is a good rule of thumb
- New table size
  - Twice-as-big is a good idea, except that won’t be prime!
  - So go *about* twice-as-big
  - Can have a list of prime numbers in your code,
    since you probably will not grow more than 20-30 times,
    and can then calculate after that

Rehashing

- What if we copy all data to the same indices in the new table?
  - Will not work; we calculated the index based on TableSize

- Go through table, do standard insert for each into new table
  - Run-time?
  - O(n): Iterate through old table

- Resize is an $O(n)$ operation, involving n calls to the hash function
  - Is there some way to avoid all those hash function calls?

  - Space/time tradeoff: Could store **`h(key)`** with each data item

  - Growing the table is still $O(n)$; only helps by a constant factor

Hashing and Comparing

- Our use of **`int`** key can lead to overlooking a critical detail
  - We initial *hash* **`E`**,
  - While chaining or probing, we *compare* to **`E`**.
    - Just need equality testing (i.e., compare == 0)

- So a hash table needs a hash function and a comparator
  - In Project 2, you will use two function objects
  - The Java library uses a more object-oriented approach: each object has an **`equals`** method and a **`hashCode`** method:

```
class Object {
  boolean equals(Object o) {…}
  int hashCode() {…}
  …
}
```

Equal Objects Must Hash the Same

- The Java library (and your project hash table) make a very important assumption that clients must satisfy
- Object-oriented way of saying it:

  If **`a.equals(b)`**, then we must require
  `a.hashCode()==b.hashCode()`
- Function object way of saying it:

  If **`c.compare(a,b) == 0`**, then we must require
  `h.hash(a) == h.hash(b)`
- If you ever override **`equals`**
  - You need to override **`hashCode`** also in a consistent way
  - See CoreJava book, Chapter 5 for other “gotchas” with **`equals`**

Comparable/Comparator Have Rules Too

We have not emphasized important "rules" about comparison for:

- all our dictionaries
- sorting (next major topic)

Comparison must impose a consistent, total ordering:

For all **`a`**, **`b`**, and **`c`**,

- If **`compare(a,b) < 0`**, then **`compare(b,a) > 0`**
- If **`compare(a,b) == 0`**, then **`compare(b,a) == 0`**
- If **`compare(a,b) < 0`** and
  `compare(b,c) < 0`, then **`compare(a,c) < 0`**

A Generally Good hashCode()

- int result = 17;
- foreach field f
  - int fieldHashcode =
    - boolean: (f ? 1: 0)
    - byte, char, short, int: (int) f
    - long: (int) (f ^ (f >>> 32))
    - float: Float.floatToIntBits(f)
    - double: Double.doubleToLongBits(f), then above
    - Object: object.hashCode( )
  - result = 31 * result + fieldHashcode

Final Word on Hashing

- The hash table is one of the most important data structures
  - Efficient **`find, insert, and delete`**
  - Operations based on sort order are not so efficient
    - e.g., **`FindMin, FindMax, predecessor`**

- Important to use a good hash function
  - Good distribution, uses enough of key's meaningful values

- Important to keep hash table at a good size
  - Prime #, preferable λ depends on type of table

- Popular topic for job interview questions
  - Also many real-world applications