CSE332: Data Abstractions

Lecture 6: Dictionaries; Binary Search Trees

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Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

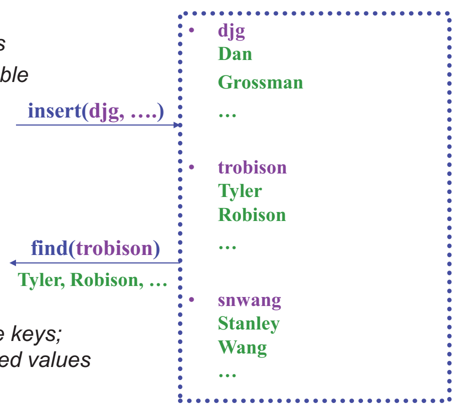
1. Stack: **`push`**, **`pop`**, **`isEmpty`**, …
2. Queue: **`enqueue`**, **`dequeue`**, **`isEmpty`**, …
3. Priority queue: **`insert`**, **`deleteMin`**, …

Next:

4. Dictionary (a.k.a. Map): associate keys with values
   – Probably the most common, way more than priority queue

The Dictionary (a.k.a. Map) ADT

- Data:
  – *set* of (key, value) *pairs*
  – keys must be *comparable*
- Operations:
  – **`insert(key,value)`**
  – **`find(key)`**
  – **`delete(key)`**
  – …



Will tend to emphasize the keys; don't forget about the stored values

Comparison: The Set ADT

The *Set* ADT is like a Dictionary without any values

– A key is *present* or not (no repeats)

For **`find`**, **`insert`**, **`delete`**, there is little difference

– In dictionary, values are "just along for the ride"
– So *same data-structure ideas* work for dictionaries and sets

But if your Set ADT has other important operations this may not hold

– **`union`**, **`intersection`**, **`is_subset`**
– Notice these are binary operators on sets

Dictionary data structures

Will spend the next several lectures implementing dictionaries with three different data structures

1. AVL trees
   – Binary search trees with *guaranteed balancing*
2. B-Trees
   – Also always balanced, but different and shallower
   – B!=Binary; B-Trees generally have large branching factor
3. Hashtables
   – Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

But first some applications and less efficient implementations…

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently

– Lots of programs do that!

- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Search: inverted indexes, phone directories, …
- Biology: genome maps
- …

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list			
• Unsorted array			
• Sorted linked list			
• Sorted array			

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$	$O(n)$	$O(n)$
• Unsorted array	$O(1)$	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\texttt{log}\ n)$	$O(n)$

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:
- Instead of actually removing the item just mark it deleted

Plusses:
- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

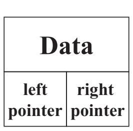
Minuses:
- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- **`find`** $O(\texttt{log}\ m)$ *time* where *m* is data-structure size (okay)
- May complicate other operations

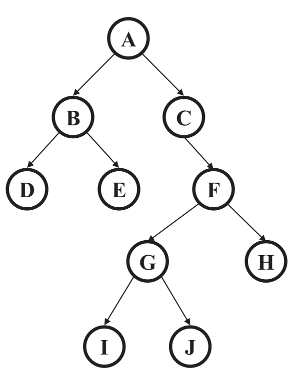
Some tree terms (mostly review)

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of "next" as the one child)
- There are many kinds of binary trees
  - Every binary min heap is a binary tree
  - Every binary search tree is a binary tree
- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of $O(\texttt{log}\ n)$
  - Different tree data structures have different "balance conditions" to achieve this

Binary Trees

- Binary tree is empty or
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*
- Representation:





- For a dictionary, data will include a key and a value

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:
- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{(h+1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(`log` n) height, and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
            ???
}
```


Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

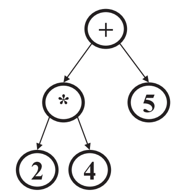
Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root

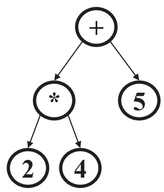


(an expression tree)

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

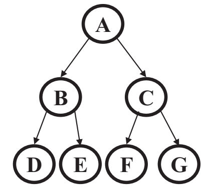
- *Pre-order*: root, left subtree, right subtree
  + * 2 4 5
- *In-order*: left subtree, root, right subtree
  2 * 4 + 5
- *Post-order*: left subtree, right subtree, root
  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



Sometimes order doesn't matter

- Example: sum all elements

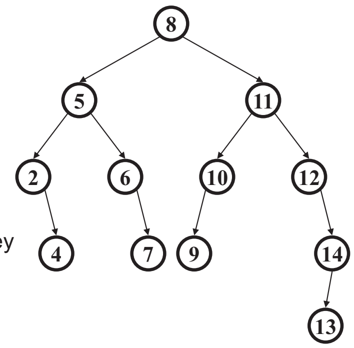
Sometimes order matters

- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

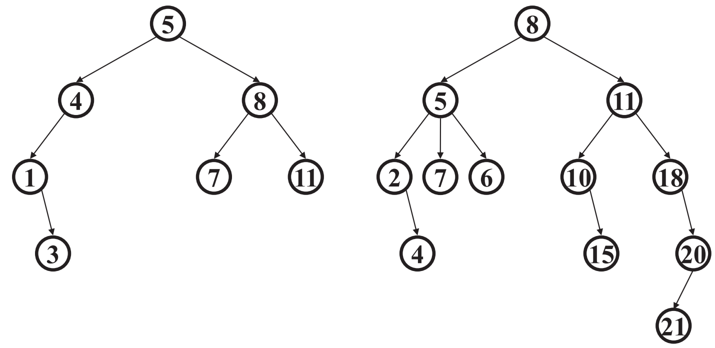
```
A
  B
    D
    E
  C
    F
    G
```

Binary Search Tree

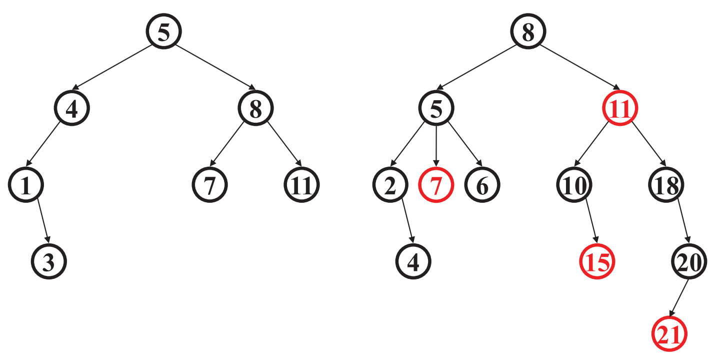
- Structure property ("binary")
  - Each node has ≤ 2 children
  - Result: keeps operations simple
- Order property
  - All keys in left subtree smaller than node's key
  - All keys in right subtree larger than node's key
  - Result: easy to find any given key



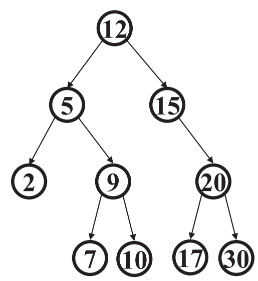
Are these BSTs?



Are these BSTs?

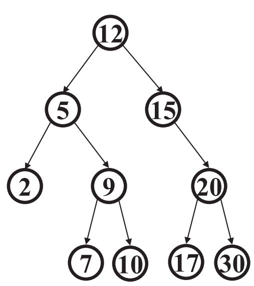


Find in BST, Recursive




```
Data find(Key key, Node root){
  if(root == null)
    return null;
  if(key < root.key)
    return find(key,root.left);
  if(key > root.key)
    return find(key,root.right);
  return root.data;
}
```


Find in BST, Iterative

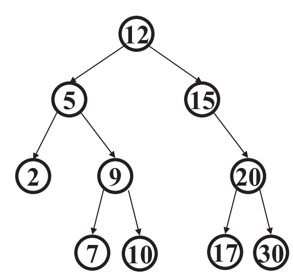



```
Data find(Key key, Node root){
  while(root != null
        && root.key != key) {
    if(key < root.key)
      root = root.left;
    else(key > root.key)
      root = root.right;
  }
  if(root == null)
      return null;
  return root.data;
}
```

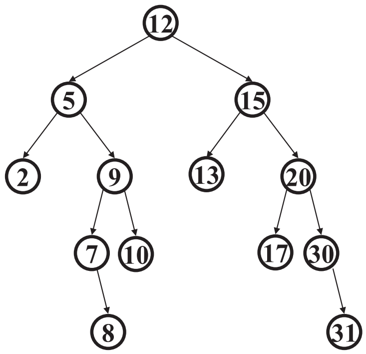

Other "Finding" Operations

- Find *minimum* node
  - "the liberal algorithm"
- Find *maximum* node
  - "the conservative algorithm"

- Find *predecessor* of a non-leaf
- Find *successor* of a non-leaf
- Find *predecessor* of a leaf
- Find *successor* of a leaf



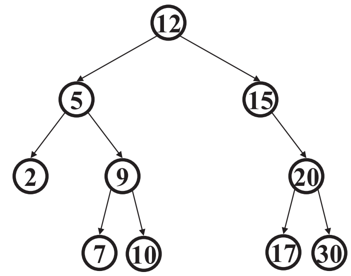
Insert in BST




```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Deletion in BST



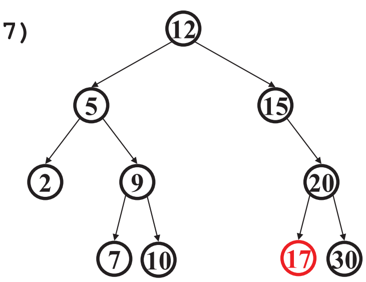
Why might deletion be harder than insertion?

Deletion

- Removing an item disrupts the tree structure
- Basic idea: `find` the node to be removed, then "fix" the tree so that it is still a binary search tree
- Three cases:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

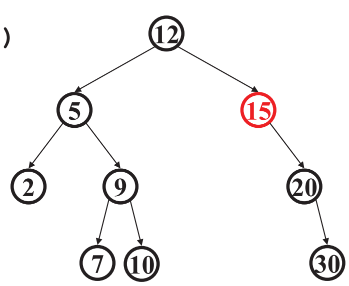
Deletion – The Leaf Case

```
delete(17)
```



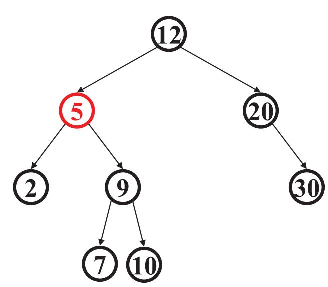
Deletion – The One Child Case

```
delete(15)
```



Deletion – The Two Child Case

```
delete(5)
```



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

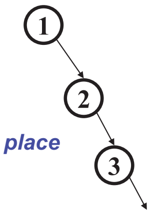
Options:
- *successor* from right subtree: `findMin(node.right)`
- *predecessor* from left subtree: `findMax(node.left)`
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*
- Leaf or one child case – easy cases of delete!

BuildTree for BST

- We had **`buildHeap`**, so let's consider **`buildTree`**
- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order, what is the tree?
  - What big-O runtime for this kind of sorted input?
  - Is inserting in the reverse order any better?

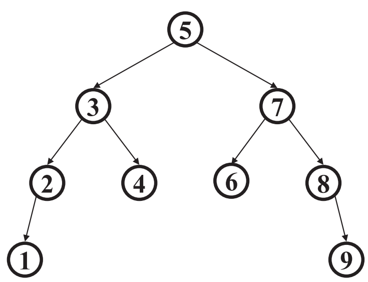
$O(n^2)$
Not a happy place



BuildTree for BST

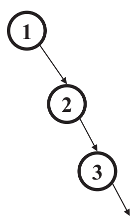
- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
- What we if could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9
  - What tree does that give us?
  - What big-O runtime?

O(n log n), definitely better



Unbalanced BST

- Balancing a tree at build time is insufficient, as sequences of operations can eventually transform that carefully balanced tree into the dreaded list
- At that point, everything is *O(n)* and nobody is happy
  - **`find`**
  - **`insert`**
  - **`delete`**



Balanced BST

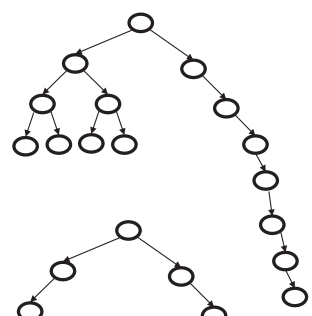
Observation
- BST: the shallower the better!
- For a BST with *n* nodes inserted in arbitrary order
  - Average height is $O(\log n)$ – see text for proof
  - Worst case height is $O(n)$
- Simple cases, such as inserting in key order, lead to the worst-case scenario

Solution: Require a **Balance Condition** that
1. Ensures depth is always $O(\log n)$ – strong enough!
2. Is efficient to maintain – not too strong!

Potential Balance Conditions

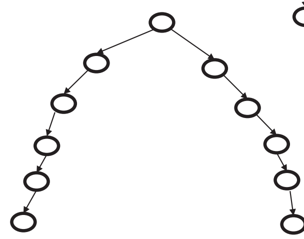
1. Left and right subtrees of the *root* have equal number of nodes

   Too weak!
   Height mismatch example:



2. Left and right subtrees of the *root* have equal *height*

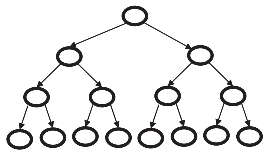
   Too weak!
   Double chain example:



Potential Balance Conditions

3. Left and right subtrees of every node have equal number of nodes

   Too strong!
   Only perfect trees (2^n – 1 nodes)



4. Left and right subtrees of every node have equal *height*

   Too strong!
   Only perfect trees (2^n – 1 nodes)

The AVL Balance Condition

Left and right subtrees of *every node* have *heights* **differing by at most 1**

Definition: **balance**(*node*) = height(*node*.left) – height(*node*.right)

AVL *property*: **for every node *x*, –1 ≤ balance(*x*) ≤ 1**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height *h* must have a number of nodes *exponential* in *h*
- Efficient to maintain
  - Using single and double rotations