1. Prove that \( n^2 \in O(2^n) \).

2. If \( T(n) \) is the run time of the code below, find a function \( f \) where \( T(n) \in \Theta(f(n)) \).

   ```
   for (i = 0; i < n; i++) {
      if (i % 2 == 0) {
         for (j = i; j < n; j++) {
            z += j;
         }
      }
   }
   ```

3. Inserting the following key into an initially empty AVL tree.

   \( 10; 20; 30; 40; 50; 22; 5; 29; 28; 27; 26; 60; 25 \)

   Then, delete the following keys.

   \( 20, 24, 25 \)

4. Perform breadth-first search on the following graph, starting from node \( a \).

   Write a list of the nodes in the order they were visited.

   ![Graph](image)

5. Insert the following keys into an initially empty Splay tree.

   \( 5, 4, 3, 100, 50, 2, 25, 10, 50, 7 \)

   Then, delete the following keys.

   \( 3, 25, 5, 2, 100 \)
6. Suppose we have a hash table using the hash function $h(x) = x \mod m$, with open hashing and linear probing.

Resize the table of size $m = 8$ below by rehashing the stored values into a table of size $m = 16$. Rehash from position 0 to $m - 1$, starting at the top of the table.

| 1 | 98 | 12 | 29 | 62 | 86 |

7. List some advantages and disadvantages of each of the dictionary implementations we’ve seen so far: unsorted lists, binary search trees, splay trees, AVL trees, and hash table.

8. Produce an ordering of the nodes $a$ through $l$ by performing topological sort on the directed acyclic graph below.

9. Run Mergesort on the following array to reorder the elements in ascending order. Show the result of each intermediate partition and merge.

| 5 | 45 | 60 | 65 | 0 | 20 | 50 | 35 | 75 | 25 | 70 | 30 |