There are 2 pages to this assignment.

1. This problem gives an orthogonal view of comparative running times from that given in Figure 2.2 of the textbook. Be sure to look at the patterns in your table when you have completed it.

For each function \( f(n) \) and time \( t \) in the following table, determine the largest size \( n \) of a problem that can be solved in time \( t \), assuming that the algorithm to solve the problem takes \( f(n) \) microseconds. For large entries (say, those that warrant scientific notation), an estimate is sufficient. For one of the rows, you will not be able to solve it analytically, and will need a calculator or small program.

<table>
<thead>
<tr>
<th>Function</th>
<th>1 second</th>
<th>1 minute</th>
<th>1 hour</th>
<th>1 day</th>
<th>1 month</th>
<th>1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1000 \log_2 n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 100n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 100n \log_2 n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10n^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n^3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{\ln} \cdot 2^n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Prove or disprove: \( n \log_{10} n \in \Theta(n \log_2 n) \).

3. (a) Prove that \( n \ln n \in O(n^{1+\epsilon}) \), for any constant real number \( \epsilon > 0 \). (Hint: choose \( c = 1 \). I see a way of doing this using derivatives, and there are probably other ways as well. If you have trouble with this, start with the case \( \epsilon = 1 \).)

(b) Prove that \( n \ln n \notin O(n) \).

4. Let \( T(n) \) be the running time of the following procedure on input \( n \). Find a function \( f(n) \) such that \( T(n) \in \Theta(f(n)) \), and justify your answer.

```plaintext
procedure triple(integer n):
    for i from 1 to n do
        for j from 3 to n/2 do
            for k from j to j + 100 do
                if \( j - k \) is even
                    then \( x \leftarrow x + 1 \);
                else \( x \leftarrow 2 \times x \);
```

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5. Let $T(n)$ be the running time of the following procedure on input $n$. Find a function $f(n)$ such that $T(n) \in \Theta(f(n))$, and justify your answer.

```plaintext
procedure double(integer n):
    for i from 1 to n do
        for j from i + 1 to n do
            x ← x + 1;
```