Name: _________________________________________

Email address: ____________________________________

Quiz Section: __________

SECTION A

CSE 326 Winter 2006: Midterm Exam
(closed book, closed notes, calculators o.k.)

Instructions Read the directions for each question carefully before answering. We will give partial credit based on the work you write down, so show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit. There is one extra page at the end of the exam that you may use for extra space on any problem. If you detach this page it must still be turned in with your exam when you leave.

Advice You have 50 minutes, do the easy questions first, and work quickly!

Total: 110 points. Time: 50 minutes. Exam ends at 1:20pm

<table>
<thead>
<tr>
<th>Question</th>
<th>Max Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>110</td>
<td></td>
</tr>
</tbody>
</table>
1. **(16 pts) Big-Oh and Run Time Analysis:**  
a. (10 points total) Describe the running time of the following pseudocode in Big-Oh notation in terms of the variable \( n \). Assume all variables used have been declared.  
*Show your work for partial credit.*

```c
int foo(int k) {
    int cost;
    for (int i = 0; i < k; ++i)
        cost = cost + (i * k);
    return cost;
}

I. answ = foo(n);
II. int sum;
    for (int i = 0; i < n; ++i) {
        if (n < 1000)
            sum++
        else
            sum += foo(n);
    }
III. for (int i = 0; i < n + 100; ++i) {
        for (int j = 0; j < i * n; ++j)
            sum = sum + j;
        for (int k = 0; k < n + n + n; ++k)
            c[k] = c[k] + sum;
    }
IV. for (int j = 4; j < n; j=j+2) {
    val = 0;
    for (int i = 0; i < j; ++i) {
        val = val + i * j;
        for (int k = 0; k < n; ++k)
            val++;
    }
}
V. for (int i = 0; i < n * 1000; ++i) {
    sum = (sum * sum)/(n * i);
    for (int j = 0; j < i; ++j)
        sum += j * i;
}
```
1. (cont.) **Big-Oh and Run Time Analysis**

   b. (6 pts) Consider the following function:

   ```
   int mystery(int n) {
       int answer;
       if (n > 0) {
           answer = (mystery(n-2) + 3*mystery(n/2) + 5);
           return answer;
       }
       else
           return 1;
   }
   ```

   Write down the complete recurrence relation, T(n), for the running time of mystery(n). Be sure you include a base case T(1). **You do not have to actually solve this relation, just write it down.**
2. (9 points) **Trees** –

a) What is the minimum and maximum number of nodes in a *complete* binary tree of height $h$?

b) What is the minimum and maximum number of nodes in a *perfect* binary tree of height $h$?

c) What is the AVL balance property?
3. (20 pts)

a.) (10 pts) Mark the following properties for each node of the tree below in the space indicated for each node: **Null Path Length (NPL)** and **Height (H)**.

![Diagram of a tree with nodes labeled 14, 7, 28, 4, 20, 18, and 23, with NPL and H indicated for each node.]

b.) (10 pts) Also, circle **yes** or **no** to indicate whether the tree above might represent each of the following data structures. If you circle **no**, give **one specific reason** why the tree could not be that data structure.

- AVL tree  
  yes  
  no

- Splay tree  
  yes  
  no

- Binary tree  
  yes  
  no

- Skew heap  
  yes  
  no

- Leftist heap  
  yes  
  no
4. (16 pts) Running Time Analysis

Give a O-bound on the worst case running time for each of the following in terms of $n$. **No explanation is required**, but an explanation may help for partial credit. Assume that all keys are distinct.

(a) insert in an *AVL tree* of size $n$

(b) insert in a *splay tree* of size $n$

(c) deleteMin in a *d-heap* of size $n$

(d) Floyd’s *buildHeap* to build a *binary heap* of $n$ elements
5. a. (8 pts) Draw the AVL tree that results from inserting the keys 4, 10, 3, 8, 5, 6, 25 in that order into an initially empty AVL tree. You are only required to show the final tree, although if you draw intermediate trees, please circle your final result for ANY credit.

b. (2 pts) Give a preorder traversal of your final AVL tree you created in part a.
c. (8 pts) Draw the Splay tree that results from inserting the keys 4, 9, 3, 7, 5, 6 in that order into an initially empty Splay tree. You are only required to show the final tree, although if you draw intermediate trees, please circle your final result for ANY credit.

You may continue your answer to this problem on the next page if needed.
d. (2 pts) Give a *postorder* traversal of your final Splay tree you created in part c.
6. **(20 pts) Heaps**

   a. (8 pts) Draw the binary heap that results from inserting 11, 9, 12, 14, 3, 15, 7, 8, 1 in that order into an **initially empty binary heap**. You do not need to show the array representation of the heap. You are only required to show the final tree, although if you draw intermediate trees, **please circle your final result for ANY credit.**
6. (cont.) **Heaps:**
b. (4 pts) Draw the binary heap that results from doing 2 deletemins on the heap you created in part a. You are only required to show the final tree, although if you draw intermediate trees, *please circle your final result for ANY credit.*
6. (cont.) **Heaps:**
c. (8 pts) Below are two leftist heaps. Show the result of merging them together using the algorithm we discussed in class. You are only required to show the final tree, although if you draw intermediate trees, *please circle your final result for ANY credit.* You may continue your answer to this question on the next page if needed.

![Diagram of two leftist heaps and their merge result](image_url)
(Extra space for Question 6. c)
7. (9 points) **Binomial Queues** –

a) What is the minimum and maximum number of nodes in a Binomial *Tree* of height $h$?

b) What is the minimum and maximum number of nodes in a Binomial *Queue* whose tallest tree is of height $h$?

c) Briefly describe how $\text{Findmin}()$ is implemented for Binomial Queues.
[Blank Page #1] Extra Space for any of the problems if needed.
[Blank Page #2] Extra Space for any of the problems if needed.