## CSE 326 Midterm - 10/26/2007

Name: $\qquad$

This exam has 10 pages, not including this title sheet. There are 8 questions worth a total of 75 points. Please budget your time appropriately.

Please write your name on every page in the space provided. Please keep everything you write for a question on the page allocated for that question (including the back of the same page if necessary). This ensures the questions can be separated from each other for efficient grading.

The exam is closed book, closed notes, etc. You may use a four function calculator if you find it helpful, but only for simple arithmetic.

Please wait to turn the page until everyone is told to begin.

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| 6 | 5 |  |
| 7 | 10 |  |
| 8 | 10 |  |
| Total | 75 |  |

Name: $\qquad$

## 1) 10 Points

## List ADT

Name two potential implementations of the List ADT that were discussed in class, and provide O (Big-Oh) bounds on the running time of each operation.

| Name | Insert <br> Average | Find <br> Average | Delete <br> Average |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Priority Queue ADT

Name an implementation of the Priority Queue ADT that was discussed in class and has the desired property. Provide O (Big-Oh) bounds on the running time of each operation.

|  | Name | Insert <br> Average | DeleteMin <br> Average | Merge <br> Average |
| :--- | :--- | :--- | :--- | :--- |
| Conceptually <br> Simple |  |  |  |  |
| Efficient <br> Merging With <br> Worst-Case <br> Guarantees |  |  |  |  |
| Efficient <br> Merging With <br> Amortized <br> Costs |  |  |  |  |
| Better Addresses <br> Costs of Caching <br> and Disk Access |  |  |  |  |

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## Dictionary ADT

Name an implementation of the Dictionary ADT that was discussed in class and has the desired property. Provide O (Big-Oh) bounds on the running time of each operation.

|  |  | Name | $\begin{array}{c}\text { Average } \\ \text { Insert }\end{array}$ | $\begin{array}{c}\text { Worst-Case } \\ \text { Insert }\end{array}$ |
| :--- | :--- | :--- | :--- | :--- | \(\left.\begin{array}{c}Worst-Case <br>

Series of k <br>
Inserts\end{array}\right]\).

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## 2) $\mathbf{1 0}$ Points

Compute an appropriately tight O (Big-Oh) bound on the running time of each code fragment. Circle your answer for each code fragment.
a)

```
for(i = 0; i < n; i++) {
    sum++;
}
```

b)

```
for(i = 0; i < n; i++) {
        for(j = 0; j < i; j++) {
        sum++;
        }
        for(k = 0; k < i; k++) {
        sum++;
        }
    }
```

c)

```
for(i = 0; i < n; i++) {
    for(j = 0; j < i * i; j++) {
        if(j % 2 == 0) {
            for(k = 0; k < i; k++) {
                sum++;
            }
        } else {
            for(k = 0; k < i * i; k++) {
                sum++;
            }
        }
        }
}
```

d)

```
for(i = 0; i < n; i++) {
    for(j = 0; j < n; j++) {
        if(i == j) {
            for(k = 0; k < i * i; k++) {
                sum++;
            }
        } else {
            for(k = 0; k < i; k++) {
                sum++;
            }
        }
    }
}
```

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## 3) $\mathbf{1 0}$ Points

Prove by induction that:
$1+4+9+\ldots+n^{2}=\frac{n(n+1)(2 n+1)}{6}$ for every positive integer $n$
If you find yourself unable to factor a polynomial, recall that you know what the polynomial should factor to. Multiplying what the factors should be and showing that the result is equal to your polynomial would be appropriate.

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## 4) $\mathbf{1 0}$ Points

A level-order traversal visits each node in a tree according to the depth of that node. For example, the nodes in this binary tree:

would be visited in the order $6,3,7,2,4,9,5$. The in-order, pre-order, and post-order traversals discussed in class are based in an implicit stack, created by the recursion. This is why I noted in lecture that you cannot perform a level-order traversal using recursion. You will write pseudocode that uses a queue to print nodes in a binary tree according to a level-order traversal.
a) Define an appropriate node type (storing an int as the data element in each node):
class Node \{
\}
b) Define an appropriate interface for a node queue (no need to worry about generics):
interface NodeQueue \{
\}

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c) Write pseudocode using your Node and NodeQueue types to implement a level-order traversal, printing the int data element from each node as it is visited.

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## 5) $\mathbf{1 0}$ Points

Consider the following binary heap:


Perform the following operations in order, drawing the resulting heap after each operation and using it as the starting point for the next operation. You need only show the result of each operation for full credit, but we will only be able to award partial credit if you show your work. If the space available here is insufficient, use the back of this sheet (clearly labeling your work). Circle the state of the heap after each operation so that we can distinguish it from your intermediate work.
a) Insert 25
b) Insert 2
c) DeleteMin
d) DeleteMin
e) DeleteMin

Name:

## 6) 5 Points

Consider the following binomial queue, currently containing 3 trees:


3

10 14

Perform a deleteMin operation on this binomial queue. You need only show the result of the operation for full credit, but we will only be able to award partial credit if you show your work. If the space available here is insufficient, use the back of this sheet (clearly labeling your work). Circle the final state of the queue so that we can distinguish it from your intermediate work.

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## 7) 10 Points

Consider the following AVL tree:


Perform the following operations in order, drawing the resulting tree after each operation and using it as the starting point for the next operation. You need only show the result of each operation for full credit, but we will only be able to award partial credit if you show your work. If the space available here is insufficient, use the back of this sheet (clearly labeling your work). Circle the state of the tree after each operation so that we can distinguish it from your intermediate work.
a) Insert 3
b) Insert 2
c) Insert 6
d) Delete 3
(We did not cover general AVL deletion, but you should know how to perform this delete)

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## 8) $\mathbf{1 0}$ Points

Consider the development of a B-tree (specifically, the variant of B-trees discussed in class and in Chapter 4). Assume that the machine your B-tree will run on stores and retrieves data in blocks of 1024 bytes and that storing a reference requires 4 bytes. Assume that you are not storing sibling references.

You will be storing personal information indexed by a person's Social Security Number (SSN). An SSN is stored in 9 bytes. The exact information being stored about each person is classified, but requires a total of 128 bytes (including space for the SSN identifying the record).

You should provide a numeric answer for questions $a, b$, and $c$, but providing the formula you used to obtain your answer will allow us to give full or partial credit if your answer is incorrect due to a simple arithmetic error.
a) Choose an appropriate value for $M$, the branching factor of internal nodes.
b) Choose an appropriate value for $L$, the number of elements per leaf.
c) What is the maximum number of records your B-tree could contain if its height is 2 ?
d) In terms of $M, L$, and $h$, what is the maximum number of records a B-tree of height $h$ might contain?

Extra Credit: In terms of $M, L$, and $h$, what is the minimum number of records a B-tree (3 points) of height $h$ must contain?

