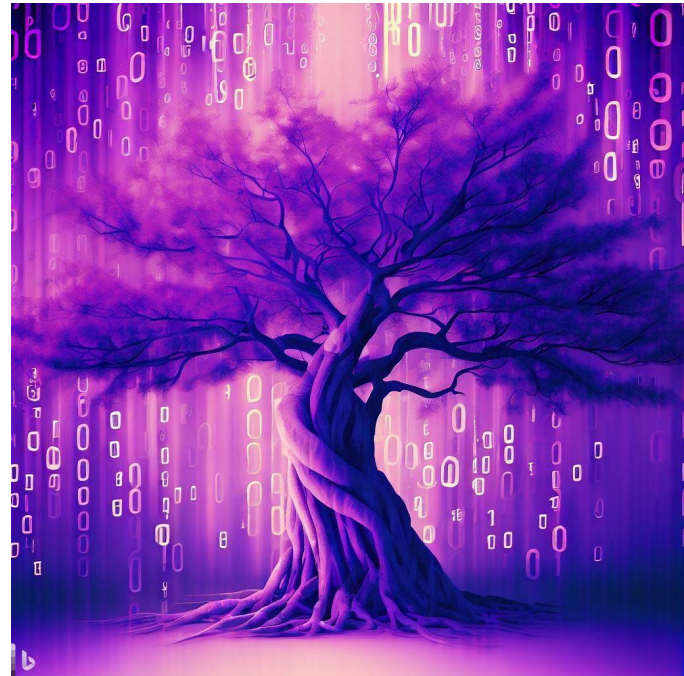


# CSE 331

## Data Types

Kevin Zatloukal



“binary tree” by DALL-E

**Correctness**

# Language Features

---

- **No need to rush out and learn all of JS / TS**
- **We will introduce language features along with the tools for reasoning about them**
- **Initially, we just need:**
  - straight-line code (const / return)
  - conditionals (if)
  - recursion
- **Will take couple weeks to learn to reason about them**

# Language Features

---

- **Next three lectures**
  1. **Data types** (data)
  2. **Functions** (code)
  3. **Proofs** (reasoning)
- **Data is the natural place to start**
  - functions operate on data, so you need data first
  - typically, the place to start when you **design** an app  
more on this later

# Language Features

---

- Reasoning is “math”
- For Data & Code, we will define
  1. Math we use to think about them
  2. How to model a specific programming language in math
- Reasoning is language independent
- Modeling is language specific
  - e.g., how to do “string or number” in Java vs TypeScript
- I will use notation to distinguish which is which

# Correctness Levels

---

Level	Description	Testing	Tools	Reasoning	
-1	small # of inputs	exhaustive			
0	straight from spec	heuristics	type checking	code reviews	amateurs
1	no mutation	“	libraries	calculation induction	} pros
2	local variable mutation	“	“	Floyd logic	
3	array / object mutation	“	“	rep invariants	

**Reasoning is what distinguishes professionals from amateurs**

# “Programming” by Trial & Error

---

- **Beginning programmers often work by trial & error**
  1. try something
  2. if that works, we're done! (fine for level -1 only)
  3. If not, go to 1
- **Easy trick to catch this: take the computer away**
  - good programmers can still function  
(can work on a programming problem at the beach!)
  - why interviews are without a computer
- **Work toward getting it right the first time**
  - carefully think through what the code is doing
  - we will work on this all quarter (starting small)

# Data Types



# Basic Data Types

---

- In math, the basic data types are “sets”
  - sets are collections of objects called **elements**
  - write  $x \in S$  to say that “x” is an element of set “S”, and  $x \notin S$  to say that it is not.

- **Examples:**

$x \in \mathbb{Z}$

x is an integer

$x \in \mathbb{N}$

x is a non-negative integer (natural)

$x \in \mathbb{R}$

x is a real number

$x \in \mathbb{B}$

x is T or F (boolean)

$x \in \mathbb{S}$

x is a character

$x \in \mathbb{S}^*$

x is a string

} non-standard names

# Basic Data Types

---

Condition	Math	TypeScript	Up to Us
integer	$x \in \mathbb{Z}$	number	no fractional part
natural	$x \in \mathbb{N}$	number	non-negative
real	$x \in \mathbb{R}$	number	
boolean	$x \in \mathbb{B}$	boolean	
character	$x \in \mathbb{S}$	string	length 1
string	$x \in \mathbb{S}^*$	string	

we will often write  
 $x : \mathbb{Z}$  instead of  $x \in \mathbb{Z}$

# Basic Data Types of JavaScript

---

- **JavaScript includes the following types**

number

string

boolean

null

undefined (another null)

Object

Array (special subtype of Object) **we won't use them until week 5/6**

- **TypeScript also includes**

unknown

any (turns off type checking — do not use!)

# Record Types

---

- JavaScript “Object” is something with “fields”
- JavaScript has special syntax for creating them

```
const p = {x: 1, y: 2};  
console.log(p.x); // prints 1
```

- The term “object” is potentially confusing
  - used for many things
  - I prefer it as shorthand for “mathematical object”
- Will refer to the math concept as a “record type”

# Type Aliases

---

- TypeScript lets you give shorthand names for types

```
type Point = {x: number, y: number};
```

```
const p: Point = {x: 1, y: 2};  
console.log(p.x); // prints 1
```

- Always include the types when declaring variables
  - otherwise, TypeScript tries to “infer” the type, and the result is sometimes not what you expect
- In math, we will do this also

```
type Point := {x:  $\mathbb{N}$ , y:  $\mathbb{N}$ }
```

# Ways to Create New Types In Math

---

- **Record Types**       $\{x : \mathbb{N}, y : \mathbb{N}\}$
- **Union Types**       $S^* \cup \mathbb{N}$ 
  - contains every object in either (or both) of those sets
- **Tuple Types**       $\mathbb{N} \times \mathbb{N}$ 
  - pair of two numbers
  - can do tuples of 3, 4, or more elements also

# Ways to Create New Types in TypeScript

---

- **Record Types**     `{x: number, y: number}`
  - anything with *at least* fields “x” and “y”
- **Union Types**     `string | number`
  - can be either one of these
- **Tuple Types**     `[number, number]`
  - at runtime, this is an array of length 2
  - should really be “`readonly [number, number]`”  
likewise for “x” and “y” in the record above

# Optional Values

---

- Records can have optional fields

```
type T = {a: number, b?: number};
```

```
const x: T = {a: 1};
```

– type of “`x.b`” is “`number | undefined`”

- Functions can have optional arguments

```
function f(a: number, b?: number): number {  
  console.log(b);  
}
```

– type of “`b`” is “`number | undefined`”



# Type Narrowing

---

- Conditionals can change the known types

```
function f(a: number, b?: number): number {
  if (b === undefined) {
    console.log("b missing 😞"); // undefined
  } else {
    console.log(2 * b);          // number
  }
}
```

- type checker “narrows” the type of “ b ” in each branch

Use “===” and “!==”  
instead of Use “==” and “!=”

# Checking Types at Run Time

---

Condition	Code
x is undefined	<code>x === undefined</code>
x is null	<code>x === null</code>
x is a number	<code>typeof x === "number"</code>
x is an integer	<code>... and Math.floor(x) === x</code>
x is a string	<code>typeof x === "string"</code>
x is an object or array	<code>typeof x === "object"</code>
x is an array	<code>Array.isArray(x)</code>

**Hard to check if x is a specific record type at runtime.  
Much easier to let the type checker do this!**

# Checking Types at Run Time

---

- Can check if a field is present using “ **in** ”
- Allows you to distinguish between two record types:

```
type T1 = {a: number, b: number};
type T2 = {c: number, b: string}

const x: T1 | T2 = ...;
if ("a" in x) {
  console.log(x.b); // number
} else {
  console.log(x.b); // string
}
```

# Structural vs Nominal Typing

---

- TypeScript uses “**structural typing**”
    - sometimes called “**duck typing**”

“if it walks like a duck and quacks like a duck, it’s a duck”
- ```
type T1 = {a: number, b: number};  
type T2 = {a: number, b: number};  
  
const x: T1 = {a: 1, b: 2};
```
- can pass “**x**” to a function expecting a “**T2**”!

# Structural vs Nominal Typing

---

- Java uses “nominal typing”

```
class T1 { int a; int b; }
```

```
class T2 { int a; int b; }
```

```
T1 x = new T1 ();
```

- cannot pass “ x ” to a function expecting a “ T2 ”

- Libraries do not interoperate unless it was pre-planned
  - create “adapters” to work around this
    - example of a design pattern used to work around language limitations

# Literal Types

---

- A literal type includes only that literal

```
const x: "red" = "red";
```

```
const y: 1 = 1;
```

- This is useful for creating small sets

```
type Color = "red" | "green" | "blue";
```

```
const c: Color = "red";
```

- Java works around this with “enums”
  - objects that “represent” red, green, and blue
- another design pattern

# Java Enums

---

- Another design pattern built into Java:

```
enum Color {  
    RED, GREEN, BLUE  
}
```

- `Color.RED` **etc.** are the only 3 instances of `Color`
- **Cannot pass a `Color` where `String` is expected**
  - must add methods to convert between them

# Inductive Data Types

---

- **Create new types using records, tuples, and unions**
  - very useful but limited
  - can only create types that are “finite” in some sense
    - if all our fields were boolean, the types would be finite sets
- **One critical element is missing: recursion**
- **Inductive data types are defined recursively**
  - combine union with recursion



# Inductive Data Types

---

- Describe a set by ways of creating its elements
  - each is a “constructor”

`type T := A(x : ℤ) | B(x : ℤ, y : T)`

- second constructor is recursive
- can have any number of arguments (even none)
  - will leave off the parentheses when there are none

- Examples of elements

`A(1)`

`B(2, A(1))`

`B(3, B(2, A(1)))`

in math, these are not function calls

# Natural Numbers

---

`type N := zero | succ(n : N)`

- **Inductive definition of the natural numbers**

|                                     |                |
|-------------------------------------|----------------|
| <code>zero</code>                   | <code>0</code> |
| <code>succ(zero)</code>             | <code>1</code> |
| <code>succ(succ(zero))</code>       | <code>2</code> |
| <code>succ(succ(succ(zero)))</code> | <code>3</code> |

**The most basic set we have is defined inductively!**

# Even Natural Numbers

---

`type  $\mathbb{E}$  := zero | two-more(n :  $\mathbb{E}$ )`

- Inductive definition of the even natural numbers

|                                                 |   |
|-------------------------------------------------|---|
| <code>zero</code>                               | 0 |
| <code>two-more(zero)</code>                     | 2 |
| <code>two-more(two-more(zero))</code>           | 4 |
| <code>two-more(two-more(two-more(zero)))</code> | 6 |

**much better notation**

# Lists

---

`type List := nil | cons(x :  $\mathbb{Z}$ , L : List)`

- **Inductive definition of lists of integers**

|                                             |                     |
|---------------------------------------------|---------------------|
| <code>nil</code>                            | $\approx []$        |
| <code>cons(3, nil)</code>                   | $\approx [3]$       |
| <code>cons(2, cons(3, nil))</code>          | $\approx [2, 3]$    |
| <code>cons(1, cons(2, cons(3, nil)))</code> | $\approx [1, 2, 3]$ |

array notation



**“Lists are the original data structure for functional programming,  
just as arrays are the original data structure of imperative programming”**



*Ravi Sethi*

**we will work with lists in HW2+ and arrays HW5+**