There are 10 questions worth a total of 100 points. Please budget your time so you get to all of the questions. Keep your answers brief and to the point.

The exam is closed book, closed notes, closed electronics, closed mouth, open mind.

Many of the questions have short solutions, even if the question is somewhat long. Don’t be alarmed.

For all of the questions involving proofs, assertions, invariants, and so forth, you should assume that all numeric quantities are unbounded integers (i.e., overflow can not happen) and that integer division is truncating division as in Java, i.e., 5/3 evaluates to 1.

If you don’t remember the exact syntax of some command or the format of a command’s output, make the best attempt you can. We will make allowances when grading.

Relax, you are here to learn.

Please wait to turn the page until everyone is told to begin.

Score _____________________ / 100
Question 1. (12 points) (Forward reasoning) Using forward reasoning, write an assertion in each blank space indicating what is known about the program state at that point, given the precondition and the previously executed statements. Your final answers should be simplified. Be as specific as possible, but be sure to retain all relevant information.

(a) (5 points)
{ a > 1 }

b = a + 3;

{ _______________________________ }

a = a + 2;

{ _______________________________ }

a = 2 * a + b;

{ _______________________________ }

(b) (7 points)
{ |y| > 2 }

if (x > 0)

{ _______________________________ }

y = y + x;

{ _______________________________ }

else

{ _______________________________ }

y = y - x;

{ _______________________________ }

{ _______________________________ }
Question 2. (12 points) (assertions) Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your final answers if possible.

(a) (5 points)
{ ________________________________ }
p = 2 * p;
{ ________________________________ }
q = p - q;
{ ________________________________ }
p = p + 1;
{ p > 0 && q > 0 }

(b) (7 points)
{ ________________________________ }
if (a < b) {
    { ________________________________ }
a = a + b;
    { ________________________________ }
} else {
    { ________________________________ }
b = b + 1;
    { ________________________________ }
}
{ a > b }
Question 3. (14 points) Consider the following binary search implementation. Answer questions about this code on the next page, but do not remove this page from the exam since you need to add assertions to this code.

```java
// return location of x in data if found, or -1 if not
// pre: data is sorted (data[0] <= data[1] <= ...)
public static int binSearch(int[] data, int x) {
    int low = 0;
    int high = data.length - 1;
    int mid = 0;
    boolean found = false;
    { inv }
    while(low <= high && !found) {
        mid = (low + high) / 2;
        if(data[mid] == x) {
            found = true;
        } else if(data[mid] < x) {
            low = mid + 1;
        } else { // data [mid] > x
            high = mid - 1;
        }
    }
    if (found) { return mid;
    } else { return -1;
    }
}
```

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Question 3. (cont) We think this code “works” since it seems to produce reasonable answers when run on some test data, but we want to be sure by proving it is correct. Without modifying the code, answer the following questions:

(a) (6 points) Give a suitable invariant for the while loop. This is an assertion that would be true at least at the point labeled with \{inv\} in the above code, also at the very beginning of the loop body, at the end of the loop body, and possibly elsewhere. The invariant should include whatever is needed to analyze the code and prove it is correct. It will need to describe the relationships between the key variables, the contents of the array, and the value x that is being searched for in the array. You may assume the array is sorted (the precondition of the method). Write the invariant below:

(b) (8 points) Prove that the binary search algorithm given on the previous page works as specified, i.e., it returns the location of x if x is present in the data array, and if it is not found returns -1. You need to add appropriate assertions in the code on the previous page. Because of the complexity of the algorithm and invariant you can be somewhat informal and don’t need to write down every intermediate assertion, but do not skip any significant steps (and, of course, don’t make misteaks).
Question 4. (10 points, 2 each) Testing. Describe five distinct black-box tests that could be used to verify that the binary search method from the previous problem works properly. Each test description should describe the test input and expected output. The input data array for your tests should be sorted properly (i.e., the appropriate precondition for the method must hold). For full credit each test should be different in some significant way from the other tests.

(a)

(b)

(c)

(d)

(e)
The next few questions concern this simple stack ADT which we would like to clean up so we can add it to one of our projects. The code works ok, but the documentation is terrible. You can remove this page from the exam for reference while working on the following questions.

```java
public class IntStack {
    private int[] vals; // stack
    private int top;    // top
    private static int defaultCapacity = 100;

    // construct new stack with default capacity
    public IntStack() {
        vals = new int[defaultCapacity];
        top = 0;
    }

    // construct new stack with given capacity
    public IntStack(int capacity) {
        vals = new int[capacity];
        top = 0;
    }

    // operations
    public boolean push(int x) {
        if (top == vals.length)
            return false;
        vals[top] = x;
        top++;
        return true;
    }

    public int pop() {
        if (top == 0) { // throw runtime exception if empty
            throw new NoSuchElementException();
        }
        top--;
        return vals[top];
    }

    public int size() {
        return top;
    }
}
```
Question 5. (12 points) (a) (3 points) Give a suitable abstract description of the class as would be written in the JavaDoc comment above the IntStack class heading.

(b) (5 points) Give a suitable Representation Invariant (RI) for this class. (Remember that this RI should be sufficient to guarantee that the existing code executes successfully.)

(c) (4 points) Give a suitable Abstraction Function (AF) for this class relating the representation to the abstract value of a IntStack.
**Question 6.** (12 points) Specification. None of the methods in the `IntStack` are specified properly. Below, supply proper JavaDoc comments for the `push` and `pop` methods for the code on previous pages, including a summary description at the beginning of each JavaDoc comment. Leave any unneeded parts blank.

```java
/**
 * @param
 * @requires
 * @modifies
 * @effects
 * @throws
 * @returns
 */
public boolean push(int x) { implementation omitted }

/**
 * @param
 * @requires
 * @modifies
 * @effects
 * @throws
 * @returns
 */
public int pop() { implementation omitted }
```
Question 7. (10 points) After your successful experience in CSE 331, you’ve been hired as a TA and a student has come in for help with a bug in their hw5 code. They have done a good job of reducing the buggy code to the following test program:

```java
import java.util.*;

class Graph {

    class Node {
        private final String label;
        Node (String label) {
            this.label=label;
        }
    }

    private Set<Node> nodes = new HashSet<Node>();

    public void addNode(String label) {
        nodes.add(new Node(label));
    }

    public int getNumNodes() {
        return nodes.size();
    }

    public static void main (String[] args) {
        Graph g = new Graph();
        g.addNode("Waffles");
        g.addNode("Pancakes");
        g.addNode("Pancakes");
        g.addNode("Maple Syrup");
        System.out.println(g.getNumNodes());
    }

}
```

The trouble is that when they run this code it prints 4, even though there should only be 3 distinct nodes in the set of nodes in the Graph.

Answer questions about this program on the next page. You should leave this page in the exam in case you need to mark corrections on the code above.

(continued on next page)
Question 7. (cont.) (a) (4 points) What’s the bug? i.e., what is the most likely reason that the main program is printing 4 when there are only three different labels on nodes in the graph? Give a brief explanation (no more than a couple of sentences).

(b) (6 points) Describe how to fix the bug. If a small amount of changed or new code is needed, or if some code should be deleted from the original program, you can show the corrections on the previous page. If more extensive additions or corrections are needed, write them below but be sure it is clear from your description where any new or changed code should be placed in the existing code. You should not make fundamental changes to the data structures or operation of the existing code – just fix it.
Question 8. (10 points) Comparing specifications. Here are four possible specifications for a method that computes whether its integer parameter \( n \) is a perfect number. (It doesn’t really matter for the question, but a perfect number is a positive integer whose value is equal to the sum of its proper positive divisors. For example, 6 is a perfect number because \( 6 = 1+2+3 \).)

Here are four possible specifications for a method that has a parameter \( n \).

A. @param n
   @returns true if \( n \) is a perfect number, otherwise false

B. @param n
   @requires \( n > 0 \)
   @returns true if \( n \) is a perfect number, otherwise false

C. @param n
   @throws IllegalArgumentException if \( n \leq 0 \)
   @returns true if \( n \) is a perfect number, otherwise false

D. @param n
   @requires \( n > 0 \)
   @returns true if \( n \) is a perfect number
   @throws RuntimeException if \( n > 0 \) but \( n \) is not a perfect number

(a) List all of the specification that are stronger than A. _________________________

(b) List all of the specification that are stronger than B. _________________________

(c) List all of the specification that are stronger than C. _________________________

(d) List all of the specification that are stronger than D. _________________________

(e) Is it possible for a single method to satisfy A and C? (yes or no) _______

(f) Is it possible for a single method to satisfy B and C? (yes or no) _______
Question 9. (6 points, 1 each) Mother’s day is this Sunday and we’ve been working on an app to help people send last-minute gifts to Mom. First, we have a class hierarchy of possible things to send:

```java
class Gift { ... }
class Flower extends Gift { ... }
class Rose extends Flower { ... }
class Violet extends Flower { ... }
```

Next, we have a basic class with a method that sends a gift:

```java
class Delivery {
    Gift send(Flower x, Flower y) { ... }
}
```

We would now like to add a new class for Mother’s day:

```java
class MomDelivery extends Delivery { ... }
```

Listed below are several possible send methods we might want to include in this new class MomDelivery. For each method circle the right choice to indicate whether it overrides the send method inherited from Delivery, or overloads it, or causes the Java compiler to indicate a type error, or is none of the previous (i.e., there are no errors, but it doesn’t overload or override the Delivery send method). Your answers should use the Java rules for subtypes and method overloading/overriding, even if true specification subtyping would give a different answer.

(a) Gift send(Rose x, Violet y) { ... }
   overload    override    error    none of the previous

(b) Flower send(Flower x, Flower y) { ... }
   overload    override    error    none of the previous

(c) Object send(Flower x, Flower y) { ... }
   overload    override    error    none of the previous

(d) Gift send(Flower x, Flower y) { ... }
   overload    override    error    none of the previous

(e) Gift send(Gift x, Gift y) { ... }
   overload    override    error    none of the previous

(f) Gift send(Flower x) { ... }
   overload    override    error    none of the previous
Question 10. (2 free points) (All reasonable answers receive the points. All answers are reasonable as long as there is an answer. 😊)

(a) (1 point) What question were you expecting to appear on this exam that wasn’t included?

(b) (1 point) Should we include that question on the final exam? (circle or fill in)

Yes

No

Heck No!!

$!@$^%$ No !!!!!

No opinion / don’t care

None of the above. My answer is __________________________.