Section 1:
Code Reasoning

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Today’s Goals

• Review of code reasoning
• Practice forward and backward reasoning on straight-line and if-statement code
• Practice identifying the strongest assertion
Before we begin . . .

• "=" vs. "=="
• Read the lecture notes
Reasoning About Code

• Two purposes
  o Prove our code is correct
  o Understand why code is correct

• Forward reasoning: determine what follows from initial conditions
• Backward reasoning: determine sufficient conditions to obtain a certain result
Worksheet

- Problems 1 through 4
- 15 Minutes – get as far as you can
- You can collaborate with other students
- Grab a TA if you feel stuck
Forward Reasoning

\{x \geq 0, \ y \geq 0\}  
y = 16;  
\{x \geq 0, \ y = 16\}  
x = x + y  
\{x \geq 16, \ y = 16\}  
x = \sqrt{x}  
\{x \geq 4, \ y = 16\}  
y = y - x  
\{x \geq 4, \ y \leq 12\}
Forward Reasoning

{true}

if (x > 0) {
    {x > 0}
    abs = x
    {x > 0, abs = x}
}
else {
    {x <= 0}
    abs = -x
    {x <= 0, abs = -x}
}
{x > 0, abs = x OR x <= 0, abs = -x}

{abs = |x|}
Backward Reasoning

\{x + 3b - 4 > 0\}
a = x + b;
\{a + 2b - 4 > 0\}
c = 2b - 4
\{a + c > 0\}
x = a + c
\{x > 0\}
Backward Reasoning

\{y > 15 \text{ || (} y \leq 5 \text{ && } y + z > 17)\}

\textbf{if} (y > 5) \{
  \{y > 15\}
  x = y + 2
  \{x > 17\}
\}\textbf{else} \{
  \{y + z > 17\}
  x = y + z;
  \{x > 17\}
\}\{x > 17\}
Implication

- Hoare triples are just an extension of logical implication
  - Hoare triple: $\{P\} S \{Q\}$
  - $P \rightarrow Q$ after statement $S$
- Everything implies true
- False implies everything

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \rightarrow Q$</th>
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Weaker vs. Stronger

• If $P_1 \rightarrow P_2$, then
  o $P_1$ is stronger than $P_2$
  o $P_2$ is weaker than $P_1$

• Weaker statements are more general
• Stronger statements are more restrictive
Worksheet

• Problem 5
Worksheet

- “I attend quiz sections.”  “I attend quiz sections on Thursdays.”
- “y > 23”  “y >= 23”
- “y = 23”  “y >= 23”
- “y < 0.00023”  “y < 0.23”
- “y is prime”  “y <= 17”
Worksheet

- “I attend quiz sections.”
- “y > 23”
- “y = 23”
- “y < 0.00023”
- “y is prime”
- “I attend quiz sections on Thursdays.”
- “y >= 23”
- “y >= 23”
- “y < 0.23”
- “y <= 17”
Worksheet

• “I attend quiz sections.”  “I attend quiz sections on Thursdays.”
• “\( y > 23 \)”  “\( y \geq 23 \)”
• “\( y = 23 \)”  “\( y \geq 23 \)”
• “\( y < 0.00023 \)”  “\( y < 0.23 \)”
• “\( y \) is prime”  “\( y \leq 17 \)”
Worksheet

- “I attend quiz sections.”
- “I attend quiz sections on Thursdays.”
- “y > 23”
- “y >= 23”
- “y = 23”
- “y >= 23”
- “y < 0.00023”
- “y < 0.23”
- “y is prime”
- “y <= 17”
Worksheet

- “I attend quiz sections.”  “I attend quiz sections on Thursdays.”
- “$y > 23$”  “$y >= 23$”
- “$y = 23$”  “$y >= 23$”
- “$y < 0.00023$”  “$y < 0.23$”
- “$y$ is prime”  “$y <= 17$”
Worksheet

• “I attend quiz sections.”

• “y > 23”

• “y = 23”

• “y < 0.00023”

• “y is prime”

“I attend quiz sections on Thursdays.”

“y >= 23”

“y >= 23”

“y < 0.23”

“y <= 17” -- ?
Weakest Precondition

• The most lenient assumptions such that a postcondition will be satisfied

• If $P^*$ is the weakest precondition for $\{P\} S \{Q\}$, then $P \rightarrow P^*$ for all $P$ that make the Hoare triple valid

• Notation: $WP = wp(S, Q)$
Weakest Precondition

\[ \text{wp}(x = y \times y, x > 4) \]
Weakest Precondition

\[ wp(x = y \times y, \ x > 4) \]
\[ |y| > 2 \]
Weakest Precondition

$wp(x = y\cdot y, \ x > 4)$

$\ |y| > 2$

$wp(y = x+1; z = y-3, \ z = 10)$
Weakest Precondition

\[
wp(x = y*y, x > 4)
\]
\[
|y| > 2
\]

\[
wp(y = x+1; z = y-3, z = 10)
\]

\[
wp(y = x+1, wp(z = y-3, z = 10))
\]

\[
wp(y = x+1, y-3 = 10)
\]

\[
wp(y = x+1, y = 13)
\]

\[
x = 12
\]
Questions