CSE 331
SOFTWARE DESIGN & IMPLEMENTATION
WORKSHEET A

Today’s Process
• If you haven’t completed the solution sheet for Worksheet A, please leave (and go finish it).
• Make sure your student ID (or name) is on your solution sheet.
• We’ll collect them all, shuffle them, and hand them out — if you get your own, let us know ASAP, since grading your own is not allowed.
• Then use a post-it to put your student ID (and name) on the sheet you are grading — otherwise we cannot give you the extra credit you should earn.

n is even \land n \leq 100
\{n = n + 2\} \land n \leq 101

1. Apply assignment rule; in this case, directly from the triple.
   - E = n+2
   - Q(n) = n is even \land n \leq 101
   - So
     - Q(E) = (n+2 is even) \land n+2 \leq 101
     - (n+2 is even) \land n \leq 99
   - The precondition is n is even \land n \leq 100
   - The precondition conjunct n is even \implies n+2 is even.
   - The precondition conjunct n \leq 100 does not \implies n \leq 99
   - So the triple is false

2. \{x = 5\} \land x = 0

3. Simplify to \false \land x = 0

4. \{System.out.println("Hello world")\} \false

5. \{x = y + 1\} \land 2x + y \leq 11

6. \{x = 5\} \land y = 0

\text{Reminder: logical implication}

A \implies B
\begin{array}{c|c|c}
A & B & A \implies B \\
\hline
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
false
{System.out.println("Hello world")} true

- See x=2Ax=3 \{ x = 5 \} x=0
- The triple is true
  - 2 points for true
  - 0 points for false
  - No partial credit

x=0 \{ while x == 0 do x = x + 1 \} x=1

- Loops of the form P\{while \& C \}Q require a loop invariant I to be defined and three properties to
  - The invariant holds on entry to the loop: P \supseteq I
  - The invariant holds after execution of the loop body: B \supseteq I
  - If the loop terminates, the postcondition can be proved: ¬B \supseteq Q
- Define I as x=0
  a. x=0 \supseteq x=0
  b. x=0 \supseteq x=0
  c. x=0 \supseteq x=0
- So the triple is true
  - 2 points for correct invariant
  - 1 point for false with correct invariant
  - 0 points for false with incorrect invariant

x=1 \{ while x != 0 do x = x + 1 \} x=100

- Define I as x\leq 100
  a. x=1 \Rightarrow x\leq 100
  b. Q(x) = x\leq 100 so Q(E) = x+1 \leq 100
     Q(E) = x=99
     Precondition x=1 \Rightarrow Q(E)
  c. x=0 \land (x\leq 100) does not \Rightarrow x=100
- Could there be an I that works? No. Negating the condition for the third part will never let x=0\land I \Rightarrow x=100 for any I
  - 2 points for false
  - 0 points for true

x=0 \{ while x == 0 do x = x + 1 \} x=1

- Loops of the form P\{while \& C \}Q require a loop invariant I to be defined and three properties to
  - The invariant holds on entry to the loop: P \supseteq I
  - The invariant holds after execution of the loop body: B \supseteq I
  - If the loop terminates, the postcondition can be proved: ¬B \supseteq Q
- Define I as x=0
  a. x=0 \supseteq x=0
  b. x=0 \supseteq x=0
  c. x=0 \supseteq x=0
- So the triple is true
  - 2 points for correct invariant
  - 1 point for false with correct invariant
  - 0 points for false with incorrect invariant

x=2 \{ if x>2 then y=1 else y=-1 \} y>\theta

- Conditionals of the form P\{if C \& S_1 else S_2 \} Q require
  (P \& C \& S_1) \land (P \& \neg C \& S_2) \land Q
- (x>2 \& x>2 \land \{ y=1 \} \land y>\theta) \land (x>2 \& x\leq 2 \land \{ y=-1 \} \land y>\theta)
- The first conjunct is true: x>2 \land \{ y=1 \} \land y>\theta
- The second conjunct is true: false \land \{ y=-1 \} \land y>\theta
- So the triple is true
  - 2 points for true
  - 0 points for false
  - No partial credit

x/y - remainder (r) and quotient (q)

true \{ r = x; q = 0; while (y \leq r) \{ r = r - y; q = 1 + q; \} \}
\{ x = r + yq \land y > r \}

- 2 for reporting that the triple is true
- 3 points for incorrect loop invariant
- 1 point for each of the three subparts that is missing
- -1 or -2 for confusing reasoning
$n > 0 \{ \\
n = 0; \\
j = 1; \\
while (k! = n) \\
{k = k + 1; \\
j = 2 \cdot j;} \\
\} \\
\j = 2^n$

$\begin{align*}
I & \equiv j = 2^n \\
A. & \ (k = 0 \land j = 1) \Rightarrow j = 2^n \\
B. & \ (k = 0 \lor j = 2^n \land (k = 1 \land j = 2^*)) \Rightarrow j = 2^n \\
C. & \ (k = 0 \lor j = 2^n) \Rightarrow j = 2^n \\
QED
\end{align*}$

-2 for reporting that the triple is false
-3 points for incorrect loop invariant
-1 point for each of the three subparts that is missing
-1 or -2 for confusing reasoning