Procedure specifications

CSE 331
Spring 2010
Outline

• Satisfying a specification; substitutability
• Stronger and weaker specifications
  – Comparing by hand
  – Comparing via logical formulas
  – Comparing via transition relations
  • Full transition relations
  • Abbreviated transition relations
• Specification style; checking preconditions
Satisfaction of a specification

• Let P be an implementation and S a specification

• $P$ satisfies $S$ iff
  – Every behavior of P is permitted by S
  – “The behavior of P is a subset of S”

• The statement “P is correct” is meaningless
  – Though often made!

• If P does not satisfy S, either (or both!) could be “wrong”
  – “One person’s feature is another person’s bug.”
  – It’s usually better to change the program than the spec
Why compare specifications?

We wish to compare procedures to specifications
  – Does the procedure satisfy the specification?
  – Has the implementer succeeded?

We wish to compare specifications to one another
  – Which specification (if either) is stronger?
  – A procedure satisfying a stronger specification can be used anywhere that a weaker specification is required
A specification denotes a set of procedures

Some set of procedures satisfies a specification
Suppose a procedure takes an integer as an argument
   Spec 1: “returns an integer ≥ its argument”
   Spec 2: “returns a non-negative integer ≥ its argument”
   Spec 3: “returns argument + 1”
   Spec 4: “returns argument²”
   Spec 5: “returns Integer.MAX_VALUE”

Consider these implementations:

<table>
<thead>
<tr>
<th>Code 1</th>
<th>Code 2</th>
<th>Code 3</th>
<th>Code 4</th>
<th>Code 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>return arg * 2;</td>
<td>return abs(arg);</td>
<td>return arg + 5;</td>
<td>return arg * arg;</td>
<td>return Integer.MAX_VALUE;</td>
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Specification strength and substitutability

• A stronger specification promises more
  – It constrains the implementation more
  – The client can make more assumptions

• Substitutability
  – A stronger specification can always be substituted for a weaker one
Procedure specifications

Example of a procedure specification:

```java
// requires i > 0
// modifies nothing
// returns true iff i is a prime number
public static boolean isPrime (int i)
```

General form of a procedure specification:

```java
// requires
// modifies
// throws
// effects
// returns
```
How to compare specifications

Three ways to compare

1. By hand; examine each clause
2. Logical formulas representing the specification
3. Transition relations
   a) Full transition relations
   b) Abbreviated transition relations

Use whichever is most convenient
Technique 1: Comparing by hand

We can **weaken** a specification by
Making **requires** harder to satisfy (**strengthening requires**)  
Preconditions: **contravariant**, all other clauses: **covariant**  
Adding things to **modifies** clause (**weakening modifies**)  
Making **effects** easier to satisfy (**weakening effects**)  
Guaranteeing less about **throws** (**weakening throws**)  
Guaranteeing less about **returns** value (**weakening returns**)  
The **strongest** (most constraining) spec has the following:  
  **requires** clause: true  
  **modifies** clause: nothing  
  **effects** clause: false  
  **throws** clause: nothing  
  **returns** clause: false  
(This particular spec is so strong as to be useless.)
Technique 2: Comparing logical formulas

Specification S1 is stronger than S2 iff:
\[ \forall P, (P \text{ satisfies } S1) \Rightarrow (P \text{ satisfies } S2) \]
If each specification is a logical formula, this is equivalent to:

\[ S1 \Rightarrow S2 \]

So, convert each spec to a formula (in 2 steps, see following slides)

This specification:

// requires R
// modifies M
// effects E

is equivalent to this single logical formula:

\[ R \Rightarrow (E \land \text{nothing but } M \text{ is modified}) \]

What about throws and returns? Absorb them into effects.

Final result: S1 is stronger than S2 iff

\[ (R_1 \Rightarrow (E_1 \land \text{only-modifies-}M_1)) \Rightarrow (R_2 \Rightarrow (E_2 \land \text{only-modifies-}M_2)) \]
Convert spec to formula, step 1: absorb **throws**, **returns**

CSE 331 style:
- requires (unchanged)
- modifies (unchanged)
- throws
- effects 
- returns 

} correspond to resulting "effects"

Example (from `java.util.ArrayList<T>`):

```java
// requires: true
// modifies: this[index]
// throws: IndexOutOfBoundsException if index < 0 || index ≥ size()
// effects: this_post[index] = element
// returns: this_pre[index]
T set(int index, T element)
```

Equivalent spec, after absorbing **throws** and **returns** into **effects**:

```java
// requires: true
// modifies: this[index]
// effects: if index < 0 || index ≥ size() then throws IndexOutOfBoundsException
// else this_post[index] = element && returns this_pre[index]
T set(int index, T element)
```
Convert spec to formula, step 2: eliminate **requires, modifies**

Single logical formula

\[ \text{requires} \Rightarrow (\text{effects} \land (\text{not-modified})) \]

“not-modified” preserves every field not in the **modifies** clause

Logical fact: If precondition is false, formula is true

Recall: \( \forall x. x \Rightarrow true; \forall x. false \Rightarrow x; (x \Rightarrow y) \equiv (\neg x \lor y) \)

Example:

// requires: true
// modifies: this[index]
// effects: E

\[ T \text{ set(int index, T element)} \]

Result:

\[ true \Rightarrow (E \land (\forall i \neq \text{index}. \text{this}_{\text{pre}[i]} = \text{this}_{\text{post}[i]})) \]
Technique 3: Comparing transition relations

Transition relation relates prestates to poststates
Contains all possible \langle\text{input, output}\rangle pairs

Transition relation maps procedure arguments to results

```java
int increment(int i) {
    return i+1;
}
```

```java
double mySqrt(double a) {
    if (Random.nextBoolean())
        return Math.sqrt(a);
    else
        return - Math.sqrt(a);
}
```

A specification has a transition relation, too
Contains just as much information as other forms of specification
Satisfaction via transition relations

A stronger specification has a smaller transition relation
Rule: P satisfies S iff P is a subset of S
(when both are viewed as transition relations)

sqrt specification ($S_{sqrt}$)
    // requires x is a perfect square
    // returns positive or negative square root
    int sqrt (int x)
    
Transition relation: $\langle 0,0 \rangle$, $\langle 1,1 \rangle$, $\langle 1,-1 \rangle$, $\langle 4,2 \rangle$, $\langle 4,-2 \rangle$, ...

sqrt code ($P_{sqrt}$)
    int sqrt (int x) {
        // ... always returns positive square root
    }

    
Transition relation: $\langle 0,0 \rangle$, $\langle 1,1 \rangle$, $\langle 4,2 \rangle$, ...

$P_{sqrt}$ satisfies $S_{sqrt}$ because $P_{sqrt}$ is a subset of $S_{sqrt}$
Beware transition relations in abbreviated form

“P satisfies S iff P is a subset of S” is a good rule
But it gives the wrong answer for transition relations in abbreviated form
(The transition relations we have seen so far are in abbreviated form!)

AnyOdd specification \((S_{\text{anyOdd}})\)

// requires \(x = 0\)
// returns any odd integer
int anyOdd (int x)

Abbreviated transition relation: \(\langle 0,1\rangle, \langle 0,3\rangle, \langle 0,5\rangle, \langle 0,7\rangle, \ldots\)

AnyOdd code \((P_{\text{anyOdd}})\)

int anyOdd (int x) {
    return 3;
}

Transition relation: \(\langle 0,3\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \langle 3,3\rangle, \ldots\)

The code satisfies the specification, but the rule says it does not
\(P_{\text{anyOdd}}\) is not a subset of \(S_{\text{anyOdd}}\)
because \(\langle 1,3\rangle\) is not in the specification’s transition relation

We will see two solutions to this problem: full or abbreviated transition relations
Satisfaction via full transition relations (option 1)

The transition relation should make explicit everything an implementation may do:

Problem: abbreviated transition relation for S does not indicate all possibilities

anyOdd specification ($S_{\text{anyOdd}}$):

// requires x = 0
// returns any odd integer

```c
int anyOdd (int x) {
    return 3;
}
```

Full transition relation: $\langle 0,1 \rangle, \langle 0,3 \rangle, \langle 0,5 \rangle, \langle 0,7 \rangle, ...$ // on previous slide
$\langle 1,0 \rangle, \langle 1,1 \rangle, \langle 1,2 \rangle, ..., \langle 1, \text{exception} \rangle, \langle 1, \text{infinite loop} \rangle, ...$ // new
$\langle 2,0 \rangle, \langle 2,1 \rangle, \langle 2,2 \rangle, ..., \langle 2, \text{exception} \rangle, \langle 2, \text{infinite loop} \rangle, ...$ // new

anyOdd code ($P_{\text{anyOdd}}$):

```
int anyOdd (int x) {
    return 3;
}
```

Transition relation: $\langle 0,3 \rangle, \langle 1,3 \rangle, \langle 2,3 \rangle, \langle 3,3 \rangle, ...$ // same as before

The rule “$P$ satisfies $S$ iff $P$ is a subset of $S$” gives the right answer for full relations

Downside: writing the full transition relation is bulky and inconvenient

It’s more convenient to make the implicit notational assumption:

For elements not in the domain of $S$, any behavior is permitted.

(Recall that a relation maps a domain to a range.)
Satisfaction via abbreviated transition relations (option 2)

New rule: \( P \) satisfies \( S \) iff \( P \mid \text{(Domain of } S) \) is a subset of \( S \)
where “\( P \mid D \)” = “\( P \) restricted to the domain \( D \)”
i.e., remove from \( P \) all pairs whose first member is not in \( D \)
(recall that a relation maps a \textit{domain} to a \textit{range})

anyOdd specification \( (S_{\text{anyOdd}}) \)
\[
// \text{ requires } x = 0
// \text{ returns any odd integer}
\]
\[
\text{int anyOdd (int x)}
\]
Abbreviated transition relation: 〈0,1〉, 〈0,3〉, 〈0,5〉, 〈0,7〉, ...

anyOdd code \( (P_{\text{anyOdd}}) \)
\[
\text{int anyOdd (int x)} \{
    \text{return 3;}
\}
\]
Transition relation: 〈0,3〉, 〈1,3〉, 〈2,3〉, 〈3,3〉, ...

Domain of \( S = \{ 0 \} \)
\( P \mid \text{(domain of } S) = \langle 0,3 \rangle \), which is a subset of \( S \), so \( P \) satisfies \( S \)
The new rule gives the right answer even for abbreviated transition relations
We’ll use this version of the notation in CSE 331
Abbreviated transition relations, summary

The abbreviated version of the transition relation can be misleading.

The true transition relation contains all the pairs when doing comparisons.

Use the expanded transition relation, or restrict the domain when comparing.

Either approach makes the “smaller is stronger rule” work.
Review: strength of a specification

A stronger specification is satisfied by fewer procedures
A stronger specification has
  – weaker preconditions (note contravariance)
  – stronger postcondition
  – fewer modifications
  Advantage of this view: can be checked by hand
A stronger specification has a (logically) stronger formula
  Advantage of this view: mechanizable in tools
A stronger specification has a smaller transition relation
  Advantage of this view: captures intuition of “stronger = smaller” (fewer choices)
Specification style

Typically have only one of effects and returns
   A procedure has a side effect or is called for its value
      Exception: return old value, as for `HashMap.put`

The point of a specification is to be helpful
   Formalism helps, overformalism doesn't

A specification should be
   – coherent (not too many cases)
   – informative (bad example: `HashMap.get`)
   – strong enough (to do something useful, to make guarantees)
   – weak enough (to permit (efficient) implementation)
Checking preconditions

Checking preconditions
– makes an implementation more robust
– provides better feedback to the client
– avoids silent errors

A quality implementation checks preconditions whenever it is *inexpensive* and *convenient* to do so