\[ \{ a^i b^j c^k \mid i = j, c = k \} \]
\[ \{ a^i b^j c^n \mid n \geq 0 \} \]
\[ \{ a^i b^n c^k \mid n \geq 0 \} \]
\[ \{ w \in \Sigma^* \mid w \notin A \} \]
\[ \forall \text{CFL } A \exists \rho \text{ such that } \forall \text{w} \in \Sigma^* \]
\[ |w| \geq \rho \text{ then } \exists \text{u,v,x,y,z} \in \Sigma^* \]
\[ \{ a \} = \{ u \cdot v \cdot x \cdot y \cdot z \} \]
\[ \forall i \geq 0 \text{ } u v_i x y \cdot z \in A \]
\[ |w| > 0 \]
\[ |u| \leq \rho \]
Lemma: A $b$-ary tree of height $h$ has $\leq b^h$ leaves.

Conversely, more than $b^h$ leaves implies height $> h$. 
Proof idea

G: a CFG for A
b = length of longest r.h.s. of a rule in G
p = b^{n+1} when n \neq 1V1, #vars in S \in L(G) with |S| \geq p

Pick a smallest parse tree for S and a longest path in that tree.

|a| \geq p \Rightarrow \exists R \in V

repeated on that path

S \Rightarrow uRv

R \Rightarrow \ast nRy

R \Rightarrow \ast x

uv \ast xy \ast z by repeating i times.
141 \geq \mu \Rightarrow \exists R \subseteq V

repeated on that path

\[ S = \bigcup_{u \in R} R^* \setminus u \neg y \]

\[ R^* \setminus x \]

\[ u \neg v \neg x \neg y \neg z \]

by repeating it times.

Why a repeat?

\[ > b^{1V1} \text{ leaves} \Rightarrow > 1V1 \text{ path length} \]

\[ \Rightarrow \text{ some variable } R \text{ repeated.} \]

Why \( N \neg y \neq z \)?

because it was smallest tree

Why \( |u \times y| \leq r \)?

pick repeat nearest leaf
Let $L = \sum a^i b^i c^i | i \geq 0 \}$

Let $p$ be count from $P \subset L$

Let $x = a^p b^p c^p$

By $P \subseteq x$ $uvxy2 \ldots$

\[\text{Since } l \leq p \]

If $a \in x$ then $uvxy = a^i b^j c^p$

Then for some $i \leq p, j \leq p$

\[i + j < 2p \]

\[\text{Sum } |uvy| > 0 \]

Case 2 no $a^i$
\[ l = 3 \quad \text{w} \text{w} \quad l \quad \text{w} \text{w} \quad 3 \]

\[ d = a^6 b a b \]

\[ d = a^6 b^7 a^6 b \]

\[ a \underbrace{a a a a a a} \quad b \underbrace{b b b b b b b b} \]

|uxyl| \leq p \quad \text{case 1; continued to at most 2 adjacent blocks of } a^6 \text{ and } b^6.

|uvxyl| \leq 2 p

u_0 \text{ removed } \text{ an } \text{ letter from left half is } k \leq p

\text{Last letter of new left half is } \text{ 1, but last of right half is } 6.

\therefore \quad \epsilon \subset L
Case 2
vxy in right half: simila

Case 3
vxy straddle middle.

u v^0 x y^0 z = a^p b^i a^j b^r

For some i ≤ p, j ≤ p

not both i = j = p

i < j too few b's

j < i ... a^i

i = j < p a^p b^i ≠ a^i b^r

"Corollary"

\{uw | u \in \{a,b\}^* \text{ not CFL} \Rightarrow \text{Java not CFL} \}

This is representative Programming languages that require variables to be declared (1st w) before use (2nd w), none of which (C, Java, C++, ++) are CFL's at this level.