"cut and paste"

\[ x \cdot y \in \mathcal{L}(M) \]

\[ x' \]

\[ \text{in reading on either } x \text{ or } x' \]

\[ x' \cdot y \\text{ must be } \in \mathcal{L}(M) \text{ too} \]
Those who cannot remember the past are condemned to repeat it.

-- George Santayana (1905) Life of Reason

**Corollary**

Every sufficiently long input string forces a DFA around a loop.

**Proof**

let \( p = |Q| \) and \( 1 \leq p \).

Let \( q_1, \ldots, q_m \) be state \( M \) in after reading \( 1 \leq i \) letter of \( w \).

By pigeonhole principle, \( \exists 0 \leq i < j \leq |W| \) at \( q_i = q_j \).
Pumping Lemma

A regular language \( L \)

\[ \exists p \quad \forall w \in L \quad |w| \geq p \Rightarrow \]

\[ \exists x, y, z \in \Sigma^* \quad s.t. \]

\[ w = xyz \]

\[ y \neq \varepsilon \]

\[ |xy| \leq p \]

\[ \forall i \geq 0 \quad xy^iz \in L \]
PL suggests all regular languages are infinite!?? Surely false...

E.g. Suppose $L = \{ a^3 \}$

PL says $\exists p \forall w \in L \mid |w| > p \Rightarrow 0 \Rightarrow 0$

Well, take $p = 2$. Then, yes indeed for all strings in $L$ of length $2r$ or greater $\exists x \in \Sigma^*$ is vacuously true, since there are no such strings in $L$.

Ditto for any finite language — $p = 1 + \max$ length string in $L$. 

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\[ L = \{ a^n b^n | n \geq 0 \} \]

If \( L \) is regular then by P.2.3

\[ \exists p \text{ and } \ldots \]

\[ w = a^p b^p \]

\[ \exists x, y, z \in \Sigma^* \]

\[ x y z = w \]

\[ |x| \geq 0 \]

\[ |x y| \leq p \]

\[ x - a^i \text{ for some } 0 \leq i < p \]

\[ y = a^j \text{ for some } 1 \leq j \leq p \]

\[ z = a^{p-i-j} b^p \]

\[ x y z = a^p + i b^p \in L \]

\[ \therefore L \text{ is not regular.} \]