Notes on Subset Construction:
1) only the top 6 states are reachable from the start state, but all 16 are required by the construction.
2) $\varepsilon$ moves come after $\Sigma$ moves. E.g.,
$$\delta'(\{q_2\}, 1) = \emptyset,$$
not $\{q_4\}$. 
Defn

\[ M_1 \ \& \ M_2 \ \text{equivalent if} \ L(M_1) = L(M_2) \]

Theorem 1.39

A nfa N is equivalent dfa M

\[ \text{given } N = (Q, \Sigma, \delta, q_0, F) \]

\[ \text{build } M = (Q', \Sigma, \delta', q_0', F') \]

(Warm up: no ε-moves) → Full version: with ε-moves

\[ Q' = 2^Q \]

\[ q_0' = \text{E}(\{q_0\}) \]

\[ F' = \{ R \subseteq Q \mid R \cap F \neq \emptyset \} \]

\[ \forall a \in \Sigma, \forall R \subseteq Q, S'(R, a) = \bigcup_{r \in R} \text{E}(S(r, a)) \]

\[ \forall R \subseteq Q, E(R) = \{ q \mid q \text{ reachable by} \]

\[ \text{or more ε-moves from some} \ r \in R \} \]
Given NFA $M$, can build one for $L(M)^*$?
Given NFA $M$, can build one for $L(M)^*$?

No (may reject)

No may accept at +1

stuff

Yes!
Given NFA $M$, can build one for $L(M)^*$?

No
May accept extra stuff

Yes!
Given NFR \( M \), can build one \( FA \ L (M)^* \)?

Yes!
I. Suppose \( x \in L_1, y \in L_2 \)
Then \( M_1 \) reading \( x \) can reach a final state, say \( q \). By construction, \( q_20 \in \Sigma(q, \epsilon) \) (where \( q_20 = \text{init} \) of \( M_2 \))
And from \( q_20 \) reading \( y \), \( M_2 \) reaches a final state. \( \therefore M \) reading \( xy \) can reach a final state, so \( xy \) accepted by \( M \)
\( \therefore L_1 \cdot L_2 \subseteq L(M) \)
For \( i = 1, 2 \), \( NFA \, M_i \), \( L_i = L(M_i) \)

\[ \begin{array}{c}
\text{For } i = 1, 2 \text{, } NFA \, M_i \text{, } L_i = L(M_i) \\
\end{array} \]

I. if \( x \in L_1 \), \( y \in L_2 \) then \( xy \in L(M) \)

II. Suppose \( w \in L(M) \)

So \( M \) reaches \( F \) reading \( w \).

But no state of \( M_1 \) is in \( F \) and only transitions between \( M_1 \) & \( M_2 \) are \( \epsilon \)-transitions from \( F_1 \) to \( F_2 \).

So, reading \( w \), \( M \) stays in \( M_1 \) a while (reading some prefix of \( w \), call it \( x \) ) then jumps from some \( q \in F_1 \) to \( F_2 \) then runs around in \( M_2 \) reading rest of \( w \) (call it \( y \) ) ending in \( F_2 = F \).

\( w = xy \) for \( x \in L_1 \) & \( y \in L_2 \)

\( L(M) \subseteq L_1 \circ L_2 \)
Regular expressions over $\Sigma$

$\emptyset$ is an r.e.

$\varepsilon$ is an r.e.

$a \ldots$ for each $a \in \Sigma$

if $R_1$ & $R_2$ are r.e.,

then so are

$(R_1 \cup R_2)$

$(R_1 \cdot R_2)$

$(R_1^*)$

The language denoted by $R$, $L(R)$

is:

$L(\emptyset) = \emptyset$

$L(\varepsilon) = \varepsilon \cup \varepsilon$

$L(R_1 \cup R_2) = L(R_1) \cup L(R_2)$
\[ L(\phi^*) = \frac{L(\phi)}{r^2}, \quad \forall \phi^* \]

**Short hands**

\[ \Sigma = \{ a, b, c \} \]

\[ L((a \cup b) \cup c) = \Sigma \]

\[ (\Sigma^* \cup \Sigma) \cdot a \]

\[ (((a \cup b) \cup c)^*) \cdot \Sigma \cdot a \]

**precedence & associativity**

\[ (a \cup b) \cup c \]

\[ a \cup b \cdot c^* \]

\[ (a \cup (b \cdot (c^*))) \]
"words ending with ".TXT" 

\[ \Sigma^* \cdot TXT \]

\[ (a \cup b \cup \ldots \cup z)^* \cdot (a \cup \ldots \cup z \cup a \cup \ldots)^* \]

\[ 2 \cdot (2 \cdot d) \]

\[ (\Sigma \Sigma)^* \]

\[ 0^* 10^* \]

\[ (\Sigma \cup \Sigma)(\Sigma \cup \Sigma) \]

\[ \Sigma \]

\[ 00 \in 0^* (10^*10^*)^* \]

\[ 00 \notin (0^*10^*10^*)^* \]

\[ (0^*10^*10^*)^* \]

\[ (d^*d^+ ud^+d^*) \cdot (\Sigma \cup E(\Sigma \cup tu-\)d^+)) \]