1. (30 points) Show that the language SEARCH = \{w\#w, x \in \{0,1\}* and w is a substring of x\} is decidable by giving an implementation level description of a decider TM M for the language. (Give your TM in the format M = “On input string s:…”; no need to give the state diagram). See examples of implementation level descriptions in Section 3.1 in the text. You may use a deterministic or nondeterministic TM with a single tape or multiple tapes.

2. (40 points: 10 each)
   i. Show that decidable languages are closed under complement.
   ii. Show that decidable languages are closed under concatenation.
  Give implementation level details of the necessary Turing machines in each case.
   iii. Can you modify your proof of (i) above to show that Turing-recognizable languages are closed under complement? Explain how. If not possible, explain why not.
   iv. Can you modify your proof of (ii) above to show that Turing-recognizable languages are closed under concatenation? Explain how. If not possible, explain why not.

3. (30 points) Let \( NO-INT_{TM} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are TMs and } L(A) \cap L(B) = \emptyset \} \). Show that \( NO-INT_{TM} \) is undecidable by giving a reduction from a known undecidable language to \( NO-INT_{TM} \). For your reduction, you may use any of the languages shown to be undecidable in Section 5.1 in the textbook (up to Theorem 5.4 and its proof; no need to read beyond this theorem for this course). (Hint: Use a proof similar to the one for Theorem 5.4 in the textbook.)

Just for fun (no points): Is the question “Does God exist?” decidable? (Hint: Assume the question has an unambiguous yes or no answer).