1. (10 points) Draw the state diagram and write down the formal description \((Q, \Sigma, \delta, q_0, F)\) of an NFA recognizing the following language:
\[ L = \{w | w \in \{0,1\}^* \text{ and } w \text{ contains both or neither 00 and 11 as substrings}\} \]

2. (10 points) Convert your NFA for \(L\) in Problem 1 above to an equivalent DFA using the "subset construction" idea we discussed in class (also described in the proof of Theorem 1.39 and Example 1.41 in the textbook). You may simplify your DFA if you wish by omitting any states that you consider unnecessary.

3. (30 points) Let \(A = \{w | w \in \{0,1\}^* \text{ and } w \text{ contains an even number of occurrences of substring 01}\}\). Let \(B = \{w | w \in \{0,1\}^* \text{ and } w \text{ ends in 101 or 011}\}\).
   a. Draw the state diagrams of NFAs recognizing \(A\) and \(B\).
   b. Draw the state diagrams of NFAs recognizing the following languages using the constructions in Theorems 1.45, 1.47, and 1.49:
      i. \(A \cup B\)
      ii. \(A \circ B\)
      iii. \(A^*\)

4. (25 points) Recall that \(x^R\) denotes the reverse of the string \(x\) (see page 14 in the textbook). Prove that if a language \(A\) is regular, then \(A^R = \{x^R | x \in A\}\) is also regular.

5. (25 points) The year is 2011. The “great recession” continues and you find yourself on the job market. Fortunately, there are still jobs available for finite-automata developers. Based on your stellar performance in 322, your instructor writes you a “walks-on-water” reference letter which lands you a job at the investment firm Moregone Stanlay. Your first assignment is to design a DFA which, when given two binary numbers (stock prices over two consecutive days in cents), decides which of the two is the larger number. Draw your DFA, write down its formal description, and explain how it solves the problem. (Hint: See Problem 1.34 on page 89 in the textbook.)