Midterm Questions

1) Are Reg langs closed under $\subseteq$?

Given $A, B \in \text{Reg}$, is $B \subseteq A$ also a Reg language?


Countercexample: $A = \Sigma^*$ (Reg)

$\Sigma^3 = \Sigma_0, 13$

Let $B = \Sigma_0^* 1^n | n \geq 0^2 = \text{not regular}$

$B \subseteq A$

2) $\Sigma^* 0^m 1^k | n, m, k \geq 0 \text{ and } m = 2n^2$

Is it regular?

$\Sigma^* 0^m 1^k | n, m, k \geq 0^3 = (000)^* 1^*$ (Reg. Expl.)

YES, IT'S REGULAR

What is another way that we can describe non-regular languages?

Example:

$B = \Sigma^* 0^n 1^n | n \geq 0^3$

$n = 0 \quad \varepsilon$

$1 \quad 01$

Each string built from

Previous one by adding a 0 to left end and a 1 to right end.

$2 \quad 0011$

$3 \quad 000111$

$4 \quad 0000111$

String $x \in B \iff x = \varepsilon$ or $x = 0y1$ where $y \in B$
**Grammar G1**

\[ S \rightarrow \varepsilon \]

\[ S \rightarrow OS1 \]  

\textbf{Rules}

Any string in the language \( B \) can be produced using these 2 rules.

"S produces ..." Alternate ways to write rules

\[ S \rightarrow \varepsilon \mid OS1 \quad \text{"OR"} \]

\textbf{Ex.}

\[ x = 000111 \quad \text{try to generate } x \text{ from the grammar} \]

\[ \text{using } S \rightarrow OS1 \]

\[ \begin{align*}
S \Rightarrow OS1 & \Rightarrow 00511 \Rightarrow 00051111 \\
& \Rightarrow 000111 \\
& \text{using } S \Rightarrow \varepsilon
\end{align*} \]

\[ S \Rightarrow 000111 \quad \text{(means } G1 \text{ generates } 000111) \]

\[ L(G) = \{ \varepsilon \mid S \Rightarrow w \} \]

\[ L(G) \text{ is the language generated by } G1 \]

Parse tree for 000111 in G1

```
      S
     / \  
    0   5 1
   /   /  
  S   S1 1
      /   /
    0   S1 1
       /     /
     5     S1
        /     /  
      0     5 1
        /     /  
      S     S1 1
         /     /  
        \     \  
         \     \  
          \     \  
           \     \  
            \     \  
             E
```

Read the generated string from leftmost leaf to rightmost leaf.

Leaves = 000111
\[ G_2 \]

\[ S \rightarrow AA \varepsilon \]

\[ A \rightarrow 1A | \varepsilon \]

\[ L(G_2) = \{ w | w \text{ contains exactly two zeroes} \} \]

\[ = 1^* 01^* 01^* \]

\[ G_3 \]

\[ S \rightarrow AA AA \]

\[ A \rightarrow 1A | 0A | \varepsilon \]

\[ L(G_3) = \Sigma^* 0 \Sigma^* 0 \Sigma^* \]

\[ L(G_4) = \{ w | w \text{ contains an even number of } 0's \} \]

\[ G_4 \]

\[ S \rightarrow AA AA S | A \]

\[ A \rightarrow 1A | \varepsilon \]

\[ L(G_4) = 1^* 0 (1^* 01^* 01^*)^* \]

\[ L(G_5) = \{ w | w \in \Sigma^* 1 \Sigma^* \text{ and } w = w^R \} \quad \text{(palindromes)} \]

\[ G_5 \]

The next thing is to write down the strings to see the
occurrence: \[ \varepsilon \quad 0 \quad 1 \] WE SEE THAT WE NEED

\[ 00 \quad 000 \quad 010 \] TO ADD A 0 TO BOTH ENDS

\[ 11 \quad 101 \quad 111 \] OR A 1 TO BOTH ENDS.
S → O S O | 1 S 1 | 0 1 | E

Formal Definition of a Grammar

Context Free Grammar (CFG)

\[ G = (V, \Sigma, R, S) \]

1. Variables = \( V \)

   (Symbols)

2. Terminals = \( \Sigma \) (Alphabet) (does not contain \( \varepsilon \))

3. Rules = \( R : \) \( V \rightarrow \text{string over } (V \cup \Sigma) \)
   \( V \rightarrow (V \cup \Sigma)^* \)

   Gives us \( \varepsilon \)

4. Start Symbol \( S \in V \)