5/24/2010

Two more weeks in quarter.
Holiday: Monday, May 31
5 more lectures including this one.
1 more homework will be assigned Friday and due on last day of class.
Final exam is on Monday of finals week.

Turing Machines (TMs)

\[ M = (Q, \Sigma, \Gamma, \delta, q_0, \text{start}, \text{accept}, \text{reject}) \]

Tape alphabet (we call it a "tape" alphabet for historical reasons)

\[ q_{\text{accept}} \rightarrow M \text{ halts and accepts input} \]

\[ q_{\text{reject}} \rightarrow M \text{ halts and rejects input} \]

Current state (deterministic machine)

Location of current read/write head

Can move left or right

Initial state

Input: Initialize by reading the start of input

Input can be used or ignored to stay in same location: just move one way, then back again

Configuration: \( q_0, 011 \)

- In state \( q_0 \)
- Reading the second 0

Final configurations:

\[ 0110, q_{\text{accept}} \]

\[ -- q_{\text{reject} --} \]
M halts IFF M enters *Acc or *REJ

INPUT w:

M on w:
- 2 outcomes
  - M HALTS $\rightarrow$ ACCEPTS
  - M HALTS $\rightarrow$ REJECTS
  - M LOOPS

M accepts w $\iff$ M reaches *Acc

M rejects w $\iff$ M reaches *REJ or M never halts

$L(M) = \frac{1}{2} |w| / M accepts w$³

L is Turing recognisable IFF $\exists$ a TM, M s.t.

$L = L(M)$

(accepts all right inputs, but might go into an infinite loop on incorrect inputs)

TM M is a "DECIDER" TM IFF M always halts for all inputs

L is decidable IFF $\exists$ decider M s.t. $L = L(M)$

State Diagrams

\[ d(q_i, a) = (q_j, b, R) \]

If you don't change the tape contents, just move to the right
Accepts any string that starts with a 1.

$L(M) = 1\Sigma^*$

$1\Sigma^*$ is Turing RECOGNIZABLE.
DECIDER $M'$:

1. $L = \Sigma^* \text{ is decidable}$

$L = \{0^n1^n0^n | n \geq 0 \}$ is not a CFL

Is $L$ decidable?

Use marker symbol "x"

Mark the 0

Move forward to first 1, mark it

Move forward to first 0, mark it

Go back to beginning and repeat

That algorithm would break on this:

010010

So need a preprocessing step to make sure it's in the right format (0's then 1's then 0's)