Midterm is closed book. Allowed one 8.5 x 11 sheet of paper, 2-sided. Typeset allowed.

Covers HW 1-4

How are regular expressions related to DFA’s / NFA’s?

**THM** \( L \text{ is regular } \iff L = L(R) \) where \( R \) is a reg. expr.

**THM** If \( R \) is a reg. expr. then \( L(R) \) is regular

i.e. \( \exists \text{ NFA } N \text{ s.t. } L(R) = L(N) \)

**PE** \( R \) is a reg. expr. Then,

1. \( R \) is a \( a \in \Sigma \)
2. \( R \) is \( \varepsilon \)
3. \( R = \theta \)
4. \( R = R_1 \cup R_2 \)
5. \( R = R_1 R_2 \)
6. \( R = R_1^* \)

\( a \in \Sigma \)

\( \varepsilon \)

\( \theta \)

\( R_1 \cup R_2 \)

\( R_1 \cdot R_2 \)

\( L(R_1) = L(N_1) \)

\( L(R_2) = L(N_2) \)
EXAMPLE

\[ R = 10 \varepsilon^* 01 = \varepsilon W_1 W \text{ starts with } 10 \text{ and ends with } 01 \]
\[ = 10 (0U1)^* 01 \]

This machine can be reduced by eliminating some states.

\[ \text{TM2} \quad L \text{ is regular} \quad \Rightarrow \quad L = L(R) \quad \text{for some reg. exp. } R. \]

\[ \text{IE} \quad L \text{ regular} \quad \Rightarrow \quad \exists \text{ DFA } M \in \mathcal{L} \quad \Rightarrow \quad L = L(M) \]

\[ \text{These are all you need to construct a reg. expr. from any DFA} \]
CONVERT DFA TO A SEQUENCE OF "GENERALIZED NFA's"
(G NFA's)

(Transitions allowed on regular expressions)

1. Combine parallel edges

2. Removal of states:

3. Add new start and final states

GOAL:

JUST 2 STATES
Example

\[ L(M) = \#(0s \text{ in } w) \text{ is not divisible by } 3 \]

1. Add start + final state, remove existing accept states.

\[ S \rightarrow q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow F \]

get rid of \( q_1 \)

\[ S \rightarrow q_0 \rightarrow q_2 \rightarrow q_1 \rightarrow q_2 \rightarrow F \]

get rid of \( q_2 \)

\[ S \rightarrow \epsilon \rightarrow q_0 \rightarrow 01^* \rightarrow F \]

\[ S \rightarrow \epsilon \rightarrow q_0 \rightarrow 01^* \rightarrow q_0 \rightarrow 01^* \rightarrow F \]

\[ S \rightarrow (01^* 01^* 0) \rightarrow F \]

\[ S \rightarrow (1U(01^* 01^* 0)) \rightarrow F \]

\[ S \rightarrow (1U(01^* 01^* 0)) \rightarrow F \]