For HW #3

1) Should be an NFA, not a DFA.

2) Don't write down all the states for the DFA, just those necessary for the transitions.

NFA

\[ N : q_0 \rightarrow q_1 \rightarrow q_2 \]

\[ L(N) = \{ w \mid w = x1a, x \in \{0,1\}^*, a \in \{0,1\} \} \]

Example

\( \omega = 11 \)

Accept 11

Convert to DFA: (Collapse the tree into a chain of states):

DFA

\( q_1, q_2 \) is not reachable, so we can ignore it.

In general, many subsets will never be used.

Any state that includes \( q_2 \) is an accept state.
THM: For every NFA $N$, there exists an equivalent DFA $M$.

$s.t. \; L(N) = L(M)$

**Pf:**

Given NFA $N = (Q, \Sigma, \delta, q_0, F)$

NFA has $k$ states

How many possible subsets of states $2^k$.

So DFA has $2^k$ possible states.

DFA $M = (Q', \Sigma, \delta', q'_0, F')$

1. $Q' = 2^Q$
2. $\Sigma' = \Sigma$
3. $q'_0 = E(q_0, \varepsilon)$
4. $F' = \{ R \in 2^Q | \exists q \in R s.t. q \in F \}$
5. $\delta'(R, a) = \{ q' | \exists q \in R s.t. \delta(q, a) = q' \}$

$E$-closure of set $A$:

$E(A): \{ q | q \text{ can be reached from a state in } A \text{ via } 0 \text{ or more } E \text{ transitions} \}$

**Example**
Cor. L is regular ⇔ ∃ DFA $M$, $L(M) = L$

⇔ ∃ NFA $N$, $L(N) = L$

Thm. If $A_1$, $A_2$ are regular, $A_1 \cup A_2$ is regular.

(Already proved via Cartesian product construction.)
Now we prove with NFAs.

Prop. NFAs $N_1, N_2$ for $A_1, A_2$.

Diagram of NFAs $N_1$ and $N_2$.
THEM: $A_1, A_2 \text{ Reg } \Rightarrow A_1 \circ A_2$ is regular

$N$

$L(N) = A_1 \circ A_2$

THERE ARE NO LONGER ACCEPT STATES