Reading Assignment: Sipser 2.2,2.3

1. Find a pushdown automaton which recognizes the language

\[ \{a^m b^n | n \leq m \leq 2n, m, n \geq 0 \} \]

You may give your answer as a state diagram (do not use the shorthand we used in class for pushing multiple symbols onto the stack in your transition function.) You need not turn in a proof of correctness, but you should right a description describing why your PDA recognizes the language.

2. (a) Convert the following CFG into a PDA

\[ S \rightarrow (S)|[(S)]|SS|\epsilon \]

For your answer you may give a state diagram, but you expand out all of the states and not use the shorthand we used in class for pushing multiple symbols onto the stack.

(b) Now, for the PDA you have constructed, show a sequence of configurations (state and stack) which would cause your PDA to accept \( ()[0[0][0]] \).

3. Give a CFG for the language recognized by the following pushdown automata:

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q1 \stackrel{\epsilon, \epsilon \rightarrow \$}{\rightarrow} q2 \stackrel{\epsilon, \epsilon \rightarrow \epsilon}{\rightarrow} q3 \stackrel{\epsilon, \$ \rightarrow \epsilon}{\rightarrow} q4 |
q5 \stackrel{\epsilon, \epsilon \rightarrow \epsilon}{\rightarrow} q6 \stackrel{\epsilon, \$ \rightarrow \epsilon}{\rightarrow} q7
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You may do this in any manner you like (i.e. you do not have to follow the method used in the book to convert PDAs to CFGs.)

4. For any language \( A \), let \( \text{PREFIX}(A) = \{ x | xy \in A \text{ for some string } y \} \). Show that the class of context free languages is closed under the \( \text{PREFIX} \) operation. Assume you are working over a fixed alphabet \( \Sigma \). (Note: the method used in class may not be the best way to prove this.)

5. Extra Credit For the glory, not for the points! Prove that any grammar for the language \( A = \{ a^i b^j c^k | i = j \text{ or } j = k \} \) must be ambiguous.