CSE 322 Winter 2008
Assignment #5

Due: Friday, February 22, 2008

Reading assignment: Read Section 2.1 of Sipser’s book and the handout on Chomsky Normal Form.

Problems:

1. Apply the state minimization algorithm to the DFA below. Show each of your steps as in the example on the minimization handout.

```
+---+---+---+---+
| s | t | u | v |
+---+---+---+---+
| p | r | q | v |
```

2. Design context-free grammars that generate each of the following languages. Justify your grammar designs.

   (a) The set \( \{ w \in \{0, 1\}^* \mid w = w^R \} \).
   
   (b) The complement of the language \( \{ a^n b^n \mid n \geq 0 \} \) in \( \{a, b\}^* \).
   
   (c) The set \( \{ w \in \{0, 1\}^* \mid w \text{ contains twice as many 1’s as 0’s} \} \).


4. In class we gave the following grammar

   \[ S \rightarrow (S) \mid SS \mid \varepsilon \]

   for the set of strings of balanced parentheses.

   (a) Show that this grammar is ambiguous.

   (b) Give a new unambiguous grammar for the same language.
5. Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar:

$$
\langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\
\langle \text{IF-THEN} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \\
\langle \text{IF-THEN-ELSE} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\
\langle \text{ASSIGN} \rangle \rightarrow a := 1
$$

$\Sigma = \{i,f,c,o,n,d,t,h,e,l,s,a,:,;,=,1\}$

$V = \{\langle \text{ASSIGN} \rangle, \langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$

$G$ is a natural-looking grammar for a fragment of a programming language, but $G$ is ambiguous.

(a) Show that $G$ is ambiguous.

(b) Give a new unambiguos grammar for $L(G)$.


7. (Extra credit) A CFG $G = (V, \Sigma, R, S)$ is regular (also known as right-linear) iff every rule of $G$ is of the form $A \rightarrow wB$ or $A \rightarrow w$ for $w \in \Sigma^*$ and $A, B \in V$. In class we showed that every regular language has a regular grammar. Show the converse, namely that for every regular grammar $G$, $L(G)$ is regular, which together with what we showed in class shows that regular grammars generate precisely the regular languages.