Reading assignment: Read sections 1.3-1.4 of Sipser’s book.

Problems:

1. For a language $L$ define the reverse of $L$, $L^R = \{x^R \mid x \in L\}$ where $x^R$ is the reverse of string $x$. Give a construction that will take a DFA $M$ that recognizes $L$ and convert it to an NFA $M'$ that recognizes $L^R$. Give a formal definition of $M'$ based on $M$ and briefly argue why your construction is correct.

2. Using the construction given by the proof of Lemma 1.55 (1st edition Lemma 1.29) (as shown in Examples 1.57 and 1.58 (1st edition 1.30 and 1.31) to draw state diagrams for NFAs that accept the languages given by the following regular expressions. Include all states that would be created by this construction.

   (a) $((ab)*b)^* \cup bb^*$
   
   (b) $(a^*b(a^*b)^*b^*)^*$
   
   (c) $(a \cup ab)^* \cup (b \cup ba)^*$

3. Sipser’s book 2nd edition Problem 1.40 (b) (1st edition Problem 1.32 (b)).


5. Use the construction given on the handout for converting NFAs to Regular Expressions to build a regular expression for the language accepted by the DFA $M_2$ in question 1.1 of Sipser’s text (both editions). Show your steps.

6. (Extra credit) Show that if $A$ is recognized by a finite automaton there is a finite automaton that recognizes the set $A_{1/2}^-$ of first halves of strings in $A$, i.e.

   $$A_{1/2}^- = \{x : xy \in A \text{ for some } y \text{ with } |x| = |y|\}.$$