Reading Assignment: Sipser 2.2,2.3

1. Find a pushdown automaton which recognizes the language

\[ \{a^m b^n | n \leq m \leq 2n, m, n \geq 0 \} \]

For the transition function, you may give a state diagram (do not use the shorthand we used in class for pushing multiple symbols onto the stack in your transition function.) You need not turn in a proof of correctness, though it would be good reassurance for yourself to do such a proof.

2. (a) Convert the following CFG into a PDA

\[ S \rightarrow (S)|[0S]|SS|\varepsilon \]

For your answer you may give a state diagram, but you expand out all of the states and not use the shorthand we used in class for pushing multiple symbols onto the stack.

(b) Now, for the PDA you have constructed, show a sequence of configurations (state and stack) which would cause your PDA to accept \( ()[0[0]|0]|0\).

3. For any language \( A \), let \( \text{PREFIX}(A) = \{ x | xy \in A \text{ for some string } y \} \). Show that the class of context free languages is closed under the \( \text{PREFIX} \) operation. Assume you are working over a fixed alphabet \( \Sigma \).

4. Convert the PDA in Figure 2.19, Page 114 in Sipser’s text (Fig 2.8, Page 106 in First edition) to a CFG.

5. Extra Credit For the glory, not for the points! Prove that any grammar for the language \( A = \{ a^i b^j c^k | i = j \text{ or } j = k \} \) must be ambiguous.

6. Extra Credit For the glory, not for the points! For a PDA \( M = (Q, \Sigma, \Gamma, \delta, q_0, F) \), we say a string \( a \in \Gamma^* \) is a possible stack of \( M \) if there is some input and some choice of moves of \( M \) such that \( a \) appears on the stack during its computation. Prove that the language \( L \subseteq \Gamma^* \) of all possible stacks of \( M \) is regular.