Reading Assignment: Lecture notes on pattern matching, Myhill-Nerode, and DFA Minimization. Sipser 2.1

Problems:

1. Use the pumping lemma to prove that the following languages are not regular:
   (a) \( L_1 = \{wwww|w \in \{a, b\}^*\} \).
   (b) \( L_2 = \{0^n1^m0^n|m, n \geq 0\} \).
   (c) \( L_3 = \{0^p|p \text{ is a prime number}\} \).

2. Use the method from the Myhill-Nerode (see lecture notes) to prove that the following languages are not regular:
   (a) \( L_4 = \{ww|w \in \{a, b\}^*\} \).
   (b) \( L_5 = \{0^n1^m0^n|m, n \geq 0\} \).
   (c) \( L_6 = \{w|w \neq w^R, w \in \{0, 1\}^*\} \). Recall \( w^R \) is the reversal of the string \( w \). So this is the language of strings which are not palindromes.

3. Show that the language
   \[ L_7 = \{a^ib^jc^k|i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\} \]

satisfies the conditions of the pumping lemma and therefore cannot be proven non-regular by the pumping lemma. Then use Myhill-Nerode (see lecture notes) to prove that the language is not regular.

4. Consider the language \( A \) of strings in \( \{a, b\}^* \) that start and end in different symbols. Describe the equivalence classes of this language.

5. Let \( C_k = \Sigma^* a \Sigma^{k-1} \) where \( \Sigma = \{a, b\} \). Prove that a DFA which recognizes \( C_k \) must have \( 2^k \) states.