CSE 322
Introduction to Formal Models in Computer Science

Sample Midterm

Closed Book, Closed Notes. Answer the problems on the exam paper. Each problem starts on a new page. Make sure you write your NAME on the paper. If you need extra space use the back of a page.

1. Answer True or False and BRIEFLY JUSTIFY your answers to the following questions. All strings considered are over some fixed alphabet $\Sigma$.
   
   (a) If $L$ is a regular language and $K \subseteq L$ then $K$ is a regular language.
   
   (b) If $M$ is a DFA and $A = L(M)$ then in general one can not build a DFA to accept $A^R$.
   
   (c) If $A$ is regular and $A \cup B$ is regular then so is $B$.
   
   (d) Let $A$ be a language over $\{0, 1\}$ and $x_0$ and $x_1$ be strings. For any $z \in A$ one can get a new string by replacing all 0’s in $z$ by the string $x_0$ and 1’s in the string by $x_1$. Let $A_{\text{substitute}}$ be the new language created from $A$ by taking all these new strings. If $A$ is regular then $A_{\text{substitute}}$ will be regular.

2. (a) Draw the state diagram of an NFA $M$ that recognizes the language $a^*b(b \cup ab)^*a$. You don’t have to use any particular construction method but you should try to avoid unnecessary states.
   
   (b) For the machine you defined in part (a), give the values of $Q, \Sigma, \delta, s, F$ where $M = (Q, \Sigma, \delta, s, F)$.
   
   (c) Show the TREE of all computation paths string $aababba$ could follow in $M$. Is it accepted by $M$? Why?

3. Build a DFA equivalent to the following NFA using the “subset construction.” You only need to show states that are reachable from the start state of your DFA (but do not attempt to simplify further).
4. In class we discussed GENERAL constructions that would take NFAs that accept languages $A$ and $B$ and produce NFAs that accept
   
   (a) $A \cup B$
   (b) $A \circ B$
   (c) $A^*$

   Apply these GENERAL constructions to the NFAs $A$ and $B$ below (just draw the state diagrams).

   ![State Diagram A](image)

   ![State Diagram B](image)

5. Let $A = \{ xx^R \mid x \in \{a, b\}^* \}$. Use the pumping lemma to prove that $A$ is not a regular language.