What's not regular?

1. $\{0^n 1^m \mid n \geq 0, m \geq 0\}$
2. $\{0, 1, 0, 1\}$
3. $\{0^n 1^n \mid n \geq 0\}$
4. $\{0, 1, 0, 1\}$

Note: $3^{rd}$ letter from right is 13.
Suppose \( G \) is characterized by \( M \) with \( k \geq 8 \) states.

Consider 8 inputs.

\[
\begin{align*}
\text{if } 00x &= 00 \\
\text{or } 01x &= 01 \\
\text{or } 10x &= 10 \\
\text{or } 11x &= 11 \\
\text{then } x &= x \\
\text{or } x &= 0 \text{ or } 1
\end{align*}
\]

By P.H.P.

2 inputs \( \rightarrow \) same stat
8. if \( 00x \)
\( 21x \)
go to same state
ERROR \( x \in \text{Fault} \)
Take any M, DFA. Suppose M has \( q \) states.

Let \( q_i \) be state M is in after reading \( 0^i \). Let \( 0 \leq i \leq \varrho \)

for any \( 0 \leq i, k \leq \varrho \), \( q_i = q_k \) if M accepts \( 0^i \)

then it also accepts \( 0^k \).

\[ L(M) = \sum_{n \geq 0^3} \frac{0^i + (n-1) \cdot 0^i}{5} \]
Take any M, DFA.
Suppose M has \( p \) states.

Let \( q_i = \text{state } M \text{ is in after random } 0^i \)
\( 0 \leq i \leq p \)

\( j \leq k \) p+1 value of \( i \), so for some
\( 0 \leq i, k \leq p \) \( q_j = q_k \) \( j \neq k \)

If \( M \) accepts \( 0^i \)

then \( q_i \) also accepts \( 0^k \).

\( i < k \).

\( \therefore L(M) = \{ 0^n \mid n \geq 0 \} \)

\( 0^j + (k-j) \cdot 0^5 \times 5 = 0^k \times 5 \)
Let $L = \exists w w^R \mid w \in \Sigma^*$

$\overline{abaabaaba}$

Suppose

Let $M$ w/ $p$ states accept $L$.

Let $w_1 \cdots w_{p+1}$ be $w$.

Let $q_i$ be state $M$ is in after reading $a^i$.

Disj.

$ba\cdots ga = gk$

$a^i b ba^i a^j b ba^i a^k ba^i a^j b ba^i a^k ba^i \cdots$ etc.

$p \geq \max\{w_i \cdots w_{p+1}\}$ all of same length

$w_j \cdot w_j \cdot R \subseteq L$

$w_k \cdot w_j \cdot R \subseteq L$
Pumping Lemma

If $L$ is regular, then there exists a positive integer $p$ such that for all $x \in L$ with $|x| \geq p$, there exist $y$ and $z$ such that $x = xyz$, $|yz| \leq p$, and for all $i \geq 0$, $xy^iz \in L$. 

$|x| \geq p$

$|xy| \leq p$

$|y| \geq 2$

$|x|y| \leq p$