Closure under \( U \)
even parity \( \leq 30,13^* \)
or 3rd from right is \( a = 30,63^* \)

\[ M_1 = (q_1, S, S_2, \theta_0, c, F) \]
\[ M = (Q_1 \times Q_2, S, S, (\theta_0, \theta_0), F) \]
\[ a \in Q_1, \ b \in Q_2 \quad a \neq b \]

\[ S \subset (B, B_2), a \]

\[ F = F_1 \times Q_1 \cup Q_1 \times F_2 \quad \text{uni} \]

\[ F' = F_1 \times F_2 \quad \text{inter} \]
To show by induction on length of $w$ A $g_1 c Q$, $g_2 c Q$. If $w$ M is in state $(q_1, q_2)$ after
Vending $w \iff M_1$ is in state $q_1$ and $M_2$ is in state $q_2$.

Car

Accepts $L(M_1) \cup L(M_2)$

* Note that there is a key difference between statement $\circ$ and the
definition of $M$: the later, via defn of $S$, says something about
the finite set of strings in $\Sigma^*$; $\circ$ talks about the infinite set of
strings in $\Sigma^*$. 
$x, y \leq 3^*$

$\exists x \cdot y = \exists x \cdot y \mid x \in X \& y \in Y$

**Examples**

$\text{Odd parity} \cdot \text{Odd parity} = \text{Even} \quad \exists \text{Odd}^*$

$\text{Odd parity} \cdot \text{Even} = \text{Odd}$

\[ A \cdot B \begin{cases} ? \quad \begin{cases} \text{Finite} \quad \text{Finite} \end{cases} \end{cases} \quad \text{possible?} \]

\[ Z^+ \cdot \emptyset = \emptyset \]

\[ x \cdot y \equiv y \cdot x \]

\[ \exists 03 \cdot \exists 13 \equiv \exists 13 \cdot \exists 03 \]

\[ \text{always true?} \]

\[ \text{no} \]

4-3
$x, y \leq 3.013$ 
$x \leq 3.031$ 
$\Rightarrow \text{"marked concatenation"}$