1. Let $x$ and $y$ be strings and let $L$ be any language. We say that $x$ and $y$ are equivalent with respect to $L$ if for every string $z \in \Sigma^*$ either $xz$ and $yz$ are both in $L$, or neither is. [So, $x$ and $y$ are not equivalent if and only if there is some string $z$ such that exactly one of $xz$, $yz$ is in $L$. Such a $z$ is said to separate $x$ from $y$ with respect to $L$.] Suppose $L$ is accepted by DFA $M$.

(a) Prove: if $M$ is in the same state after reading $x$ as it is after reading $y$, then $x$ and $y$ are equivalent with respect to $L$.

(b) Give an example showing that the converse of statement (1a) above is false.

(c) Prove: if there are $k$ strings $\{x_1, \ldots, x_k\}$ no two of which are equivalent with respect to $L$, then $M$ has at least $k$ states. [Hint: pigeon hole principle.]

(d) In lecture I sketched a $2k + 2$ state DFA accepting the language $L_k = \{a^n b^n|1 \leq n \leq k\}$. Prove that any DFA accepting $L_2$ must have at least 6 states.

(e) [Extra Credit:] Prove that $L_k$ requires at least $2k + 2$ states for each $k \geq 1$.

(f) [Extra Credit:] Extend the idea in part (c) to give another approach to proving that a given language is not regular. Use it to prove that $L = \{a^n b^n|1 \leq n\}$ is not regular.

2. 1.29b (1st ed.: not present)

3. 1.30 (1st ed.: 1.18)

4. Let $\Sigma = \{a, b\}$.

(a) Prove that $G = \{w \in \Sigma^*|w$ is a palindrome$\}$ is not regular.

(b) Prove that $F = \{w \in \Sigma^*|w$ is not a palindrome$\}$ is not regular. [Hint: see exercise 1.14 (1st ed.: 1.10).]

5. 1.54 (1st ed.: not present)