Homework Assignment # 5

Due Date: Friday, Feb 24 (at the beginning of class)

1. (10 points: 5 points each) Consider the CFG $G_4$ in Exercise 2.1 in the textbook (both editions). Give parse trees and leftmost derivations for the following strings:
   a. $( ( a ) \times a )$
   b. $a + ( a \times ( a + a ) )$

2. (35 points: 7 points each) Let $\Sigma = \{0,1\}$. Give CFGs that generate the following languages over $\Sigma$:
   a. $\{ w | w \text{ contains the substring 10 and ends in } 0 \}$
   b. $\{ w | w \text{ contains an odd number of 0's and at least two 1's} \}$
   c. the set of all strings except the empty string and the string 0
   d. $\{1^i0^j1^k | i, j \geq 1 \text{ and } k = i + j \}$
   e. $\{ w | w = w_1w_2 \text{ where } w_1, w_2 \in \Sigma^*, |w_1| = |w_2| \text{ and } w_1 \neq w_2 \}$

3. (15 points: 5 points each) Show that context-free languages are closed under the following operations:
   a. concatenation
   b. string reversal
   c. Suffix, where for any language $L$, $\text{Suffix}(L) = \{ y | y \in \Sigma^* \text{ and } xy \in L \text{ for some string } x \in \Sigma^* \}$

4. (20 points: 10 points each) Let $\Sigma = \{0,1\}$.
   a. Show that the following CFG is ambiguous:
      $$S \rightarrow ABA \quad A \rightarrow 0A \mid \epsilon \quad B \rightarrow 1B \mid \epsilon$$
   b. Give an equivalent unambiguous CFG.

5. (20 points: 10 points each) Give informal descriptions (as in Example 2.16 in the textbook (2.10 in the 1st ed.)) and state diagrams of pushdown automata (PDA) for the following languages over $\Sigma = \{0,1\}$:
   a. $\{ w | \text{the number of 0s in } w \text{ is two times the number of 1s in } w \}$
   b. $\{0^i1^j0^k | i = j \text{ or } j = k \}$