Diagonalization

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May 31

Announcements

- Pick up solutions to H/W #7
- Pick up old graded H/Ws
- Official feedback at the end of class today
- Reminders
  - Final exam
    - Monday, June 5, 2:30-4:20p, here
  - Review Session
    - Sunday, June 4th, 3:00-4:00p, TBA

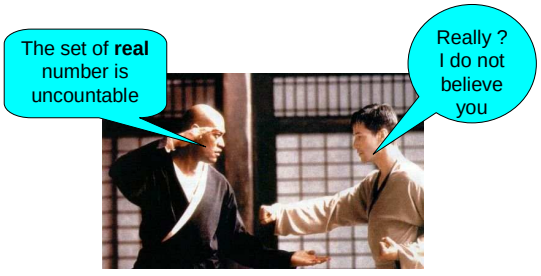
Today's puzzle

- Prove that Σ^* is countable
  - Σ is fixed, say $\{0,1\}$
- Prove that the set of all languages $L \subseteq \Sigma^*$ is uncountable

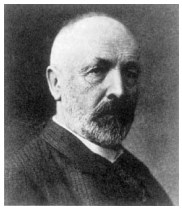
Last lecture

- A set B is countable if
  - There exists a function $f : \mathbb{N} \rightarrow B$
  - f is one to one and onto

In an alternate matrix…



But Morpheus knows Cantor…



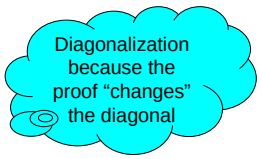
Cantor's proof

- For contradiction, assume $\mathbb{R}$ is countable
- Exists a one to one onto function $f:\mathbb{N} \rightarrow \mathbb{R}$
- List for each i, the real f(i)
- "Construct" a real r
  - There no i such that r = f(i)

1	000.12345…
2	009.19090…
3	100.00000…
4	000.56792…
⋮	

Construction of the absent r

- $r = 0.r_1r_2r_3r_4\ldots$
- $r_i \neq$ ith digit of f(i)
- For every i, $r \neq f(i)$
  - Contradiction as f is onto



1	000.12345…
2	009.19090…
3	100.00000…
4	000.56792…
	000.45 73…

Up next…

- $A_{TM} = \{ \langle M,w \rangle \mid M \text{ is a TM that accepts } w \}$
- This language is undecidable
  - Proof by diagonalization
  - First "version" might seem like "magic"
- "Deciding if a TM is a decider is undecidable"

Feedback

- If you are writing comments on the yellow sheet
- It would be great if in the improvement section, mention
  - Anything you think is important
  - Coverage of topics
    - Something you wish you had seen more/less of ?
  - Puzzles
  - Neighbor talk time