Using Pumping lemma for CFLs

Atri Rudra

May 19

Announcements

- Turn in your H/W #6
- Sorry, no graded H/W #5’s today
  - Will be handed out in class on Monday
- Take a copy of H/W #7
- Take a copy of solutions to H/W #5
  - If you did not last class

A few words on the feedback

- Thank you
- Microphone does not work properly (?)
- Solutions to sample final
  - Will try and give solution sketches
- Instructor speaks too fast
- Proofs without examples are useless
  - How to use a proof?
- Sometimes get lost in long proofs
  - What does a small step achieve in the big picture

Puzzle for today

- Prove that the following language is not a CFL
  - \{ a^n | n \text{ is prime} \}

Statement of the pumping lemma

- If \( L \) is a CFL then
  - \( \exists \) integer \( p \geq 1 \)
  - \( \forall \) strings \( s \in L \) with \( |s| \geq p \)
  - \( \exists \) strings \( u,v,x,y,z \) satisfying \( s = uvxyz \) with
    - \( |xy| \leq p \)
    - \(|v| > 0 \) or \(|y| > 0 \)
  - \( \forall \) integer \( i \geq 0 \), \( uv^ixy^iz \in L \)

The “proof”

- \( L \) is CFL
- \( L \) is accepted by a grammar \( G \)
- Consider any string “long enough” \( s \) in \( L \)
  - The parse tree must have a repeated variable
- Repeating the derivation between the repeats will give new strings that are also in \( L \)
Contrapositive of the pumping lemma

- If $L$ is a CFL then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\exists$ strings $u,v,x,y,z$ satisfying $s=uvxyz$ with
    - $|vxy| \leq p$
    - $|v| > 0$ or $|y| > 0$
  - $\exists$ integer $i \geq 0$, $uv^i xy^i z \not\in L$
  - then $L$ is not CFL

Using the pumping lemma

- If $L$ is a CFL then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\forall$ strings $u,v,x,y,z$ satisfying $s=uvxyz$ with
    - $|vxy| \leq p$
    - $|v| > 0$ or $|y| > 0$
  - $\exists$ integer $i \geq 0$, $uv^i xy^i z \not\in L$
  - then $L$ is not CFL

Let's start the duel with the devil

- $\{ a^n b^n c^n \mid n \geq 0 \}$

What have we done till now?

- Worked with the Simpson parents

Now we move onto

- Turing machines (aka Lisa Simpson)

In terms of machines

- DFA
- PDA = DFA + stack
- TM = DFA + $\infty$ memory
A quick question

- Which model represents a PC?
- Depends on how “faithful” representation you want
- No PC has infinite memory
- But for “practical” purposes it is “infinite”