Pumping Lemma

Atri Rudra
May 17

1. (d) on H/W #6
- Should read exactly one extra (Problem say one extra)

Last lecture
- Converting PDAs to CFG
- Given \( M = (Q, \Sigma, \Gamma, \delta, s, \{f\}) \)
- Build a \( G = (V, \Sigma, R, A_s) \)
  - \( V = \{ A_{pq} | p, q \in Q \} \)
- Three kinds of rules
  - For every \( p, q, r, s \in Q, t \in \Gamma, a, b \in \Sigma \cup \{\epsilon\} \), if
    \[
    a \cdot x \rightarrow t
    \]
    \[
    s \xrightarrow{b, t} \epsilon
    \]
  - then add the rule \( A_{pq} \rightarrow aA_{rs}b \)

The other rules
- For every \( p, q, r \in Q \)
  - \( A_{pq} \rightarrow A_pA_{rq} \)
- For every \( p \in Q \)
  - \( A_{pp} \rightarrow \epsilon \)
- Recall \( A_{pq} \Rightarrow^* w \) "signifies" the following:
    \[
    \text{p.} \quad \text{w} \quad \text{q.}
    \]
Idea (hope?) behind the rules

- \[ A_{pq} \rightarrow aA_{rs}b \]
- \[ \mathcal{L} \]

Why does this work?

- We need a proof...

- Today briefly sketch one direction
- If \[ A_{pq} \] generates a string then
- \[ \mathcal{L} \]
- See Claim 2.30, 2.31 in Sipser (Pgs 121-122)
- Formal proof by induction for both directions

The induction hypothesis

- For any \( p,q \in \mathcal{Q} \), if \( A_{pq} \Rightarrow^* w \) in \( \leq k \) steps then
- \[ \mathcal{L} \]
- Inductive step: \( A_{pq} \Rightarrow^* w \) in \( k+1 \) steps
  - The derivation has to look like either
  - \( A_{pq} \rightarrow aA_{rs}b \Rightarrow^* ayb=w \), or
  - \( A_{pq} \rightarrow A_{pr}A_{rq} \Rightarrow^* yz=w \)

Case 1: \( A_{pq} \rightarrow aA_{rs}b \Rightarrow^* ayb=w \)

- \( A_{rs} \Rightarrow^* y \) takes \( k \) steps
  - By induction hypothesis
  - This also implies

\[ \mathcal{L} \]

\[ \mathcal{L} \]
Case 2: $A_{pq} \rightarrow A_{pr} A_{rq} \Rightarrow^* yz = w$

- By induction
  
  $p, \overset{y}{\cdots} r, \overset{z}{\cdots} q$
  
- Thus,
  
  $p, \cdots r, \cdots q$

Questions?

Up next: Ms. PDA finds out her limits

And we will see our friend…

(Another) pumping lemma