Pumping Lemma

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May 17

Announcements

- Handouts
  - Feedback form
    - Last “unofficial” feedback
  - Solutions to H/W #5
- Office hours next week
  - Monday, May 22nd : no office hours
  - Wednesday, May 24th : 3:40-4:30pm
- No puzzle today

1 (d) on H/W #6

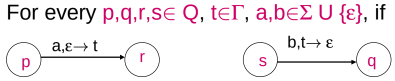
- Should read exactly one extra (
  - Problem say one extra)

Jenny’s office hours

- No office hour today
- Tomorrow, Thu May 18, 1:00 – 3:00pm
- 4th floor breakout area

Last lecture

- Converting PDAs to CFG
- Given $M = \langle Q,\Sigma,\Gamma,\delta,s,\{f\}\rangle$
- Build a $G = \langle V,\Sigma,R,A_{sf}\rangle$
  - $V = \{A_{pq} \mid p,q \in Q\}$
- Three kinds of rules
  - For every $p,q,r,s \in Q$, $t \in \Gamma$, $a,b \in \Sigma \cup \{\varepsilon\}$, if



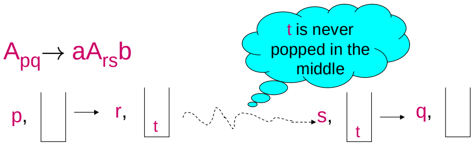
  - then add the rule $A_{pq} \rightarrow aA_{rs}b$

The other rules

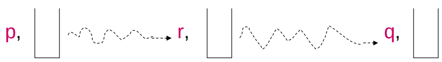
- For every $p,q,r \in Q$
  - $A_{pq} \rightarrow A_{pr}A_{rq}$
- For every $p \in Q$
  - $A_{pp} \rightarrow \varepsilon$
- Recall $A_{pq} \Rightarrow^* w$ “signifies” the following:

Idea (hope ?) behind the rules

- $A_{pq} \rightarrow aA_{rs}b$



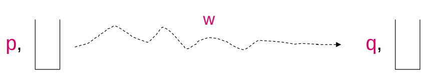
- $A_{pq} \rightarrow A_{pr}A_{rq}$



Why does this work ?

We need a proof…

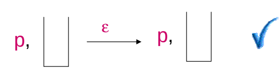
- Today briefly sketch one direction
- If A_{pq} generates a string then



- See Claim 2.30, 2.31 in Sipser (Pgs 121-122)
  - Formal proof by induction for both directions

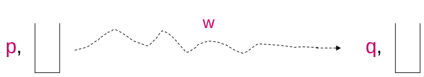
$A_{pq} \Rightarrow^* w$

- Induction on number of steps in the derivation
- Base case: 1 step in the derivation
  - Only possible rule is $A_{pp} \rightarrow \varepsilon$



The induction hypothesis

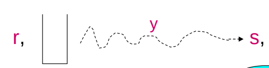
- For any $p,q \in Q$, if $A_{pq} \Rightarrow^* w$ in $\leq k$ steps then



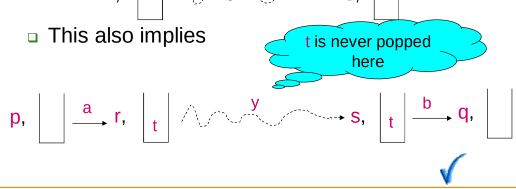
- Inductive step: $A_{pq} \Rightarrow^* w$ in $k+1$ steps
  - The derivation has to look like either
  - $A_{pq} \rightarrow aA_{rs}b \Rightarrow^* ayb=w$, or
  - $A_{pq} \rightarrow A_{pr}A_{rq} \Rightarrow^* yz=w$

Case 1: $A_{pq} \rightarrow aA_{rs}b \Rightarrow^* ayb=w$

- $A_{rs} \Rightarrow^* y$ takes k steps
  - By induction hypothesis



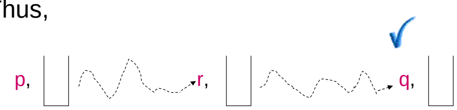
  - This also implies



Case 2: $A_{pq} \to A_{pr}A_{rq} \Rightarrow^* yz=w$

- By induction

- Thus,



Questions ?

Up next: Ms. PDA finds out her limits

And we will see our friend…

- (Another) pumping lemma