CSE322: Formal Models in Computer Science Spring 2006

PROBLEM SET 8

Due Friday, June 2, 2006, in class

Reading Assignment: Read Sections 4.1 and 4.2 of Sipser's text.

Instructions: The basic instructions are the same as in Problem Set 1.

There are **SIX** questions in this assignment. The problems in this homework are not hard per se but given that we have studied Turing Machines only for a short time, you *might* find the problems here hard. In either case, no harm in starting early. Also **do not forget** to mention the names of your collaborators in your homework.

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1. ($5 \times 3 = 15$ points) In this problem, we will look at closure properties of decidable and Turing-recognizable languages.

   (a) Show that decidable languages are closed under intersection and complement.
   (*Hint*: Take a look at the solution to problem 3.15(a) in Sipser).

   (b) Show that Turing-recognizable languages are closed under intersection.
   (*Hint*: Take a look at the solution to problem 3.16(a) in Sipser).
   Note that the complement is absent in the above. We will hopefully see an example in class where the complement of a Turing-recognizable language is not Turing-recognizable.

2. (10 points) Show using a proof by diagonalization that the set of all infinite binary sequences over $\{0, 1\}$ is uncountable. (Recall that since all strings have finite length, infinite binary sequences are not strings).

3. (10 points) Let $T = \{(i, j, k) \mid i, j, k \in \mathbb{N}\}$, where $\mathbb{N}$ is the set of natural numbers. Show that T is countable.

4. (12 points) Let $ALL_{DFA} = \{\langle D \rangle \mid D \text{ is a DFA and } L(D) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

   (*Hint*: Theorem 4.4 in Sipser might be helpful.)

5. (13 points) Let $A_{\epsilon CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \epsilon\}$. Show that $A_{\epsilon CFG}$ is decidable.

6. (**Bonus**) (10 points) Let C be a language. Prove that C is Turing-recognizable if and only if there exists a decidable language D such that $C = \{x \mid \exists y(\langle x, y \rangle \in D)\}$.