CSE322: Formal Models in Computer Science Spring 2006

PROBLEM SET 5

Due Friday, May 12, 2006, in class

Reading Assignment: Finish reading Sec 2.1 of Sipser and handouts on Chomsky Normal form and CKY algorithm.

Instructions: The basic instructions are the same as in Problem Set 1.

There are **SEVEN** questions in this assignment. This is probably a long problem set, so start early.

Do not forget to mention the names of your collaborators in your homework.

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1. (*) (10 points) Consider the DFA $M = \langle\{a, b, c, d, e, f, g, h, i\}, \{0, 1\}, \delta, a, \{c, f, i\}\rangle$, where the transition function δ is given by the following table:

	0	1
a	b	e
b	c	f
c	d	h
d	e	h
e	f	i
f	g	b
g	h	b
h	i	c
i	a	e

Using the method covered in class find the minimum-state DFA that is equivalent to M. Show your steps.

2. (*) ($3 \times 5 = 15$ points) Show that context-free languages are closed under the union, concatenation and star operators. (You do not need to *prove* that your construction works but informally argue why your construction works.)

3. ($4 \times 10 = 40$ points) Give context free grammars for the following languages and brief explain why your grammar works.

   (a) The complement of the language $\{a^n b^n \mid n \geq 0\}$ (the alphabet is $\{a, b\}$).

   (b) $\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1$, each $x_i \in \{a, b\}^*$, and for some i and j, $x_i = x_j^R\}$. (The alphabet is $\{a, b, \#\}$.)
   (Hint: It might be easier to first think about the following language $\{w \# x \mid w^R$ is a substring of x for $x, w \in \{a, b\}^*\}$.)

   (c) (*) The set of strings over $\{0, 1\}$ with thrice as many 1's as 0's.

   (d) (*) $\{R \mid R$ is a regular expression over $\{0, 1\}\}$.

4. ($2 \times 10 = 20$ points) In this problem we will consider the Chomsky Normal form of context free grammars.

   (a) Let G be an arbitrary grammar in Chomsky Normal form. Prove that any string $w \in L(G)$ of length $n \geq 1$ has exactly $2n - 1$ steps in its derivation.

   (b) (*) Convert the following grammar into Chomsky Normal form. Show all your steps.

$$
\begin{aligned}
S &\rightarrow aAa \mid bBb \mid \epsilon \\
A &\rightarrow C \mid a \\
B &\rightarrow C \mid b \\
C &\rightarrow CE \mid BCD \mid \epsilon \\
D &\rightarrow A \mid B \mid ab
\end{aligned}
$$

5. ($5 + 10 = 15$ points) Consider the following grammar $PROG - FRAG = (V, \Sigma, R, \langle ST \rangle)$ for a fragment of a programming language:

$$
\begin{aligned}
V &= \{\langle ST \rangle, \langle ASSIGN \rangle, \langle IF - THEN \rangle, \langle IF - THEN - ELSE \rangle\} \\
\Sigma &= \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a} = \texttt{1;}\},
\end{aligned}
$$

and $PROG - FRAG$ has the following rules:

$$
\begin{aligned}
\langle ST \rangle &\rightarrow \langle ASSIGN \rangle \mid \langle IF - THEN \rangle \mid \langle IF - THEN - ELSE \rangle \\
\langle IF - THEN \rangle &\rightarrow \texttt{if condition then } \langle ST \rangle \\
\langle IF - THEN - ELSE &\rightarrow \texttt{if condition then } \langle ST \rangle \texttt{ else } \langle ST \rangle \\
\langle ASSIGN \rangle &\rightarrow \texttt{a} = \texttt{1;}
\end{aligned}
$$

   (a) (*) Show that $PROG - FRAG$ is ambiguous.

   (b) Give a new unambiguous grammar that generates the same language as $PROG - FRAG$. You do not have to *prove* unambiguity, but informally describe why your new grammar is not ambiguous.

6. (**Bonus**) (5 points) Sipser's text, page 130, Problem 2.19 (Pg. 122, Problem 2.25 in first edition). Justify your description of $L(G)$.

7. (**Bonus**) (10 points) A CFG $G = (V, \Sigma, R, S)$ is *right-linear* if and only if every rule in R is of the form $A \rightarrow wB$ or $A \rightarrow w$ for $w \in \Sigma^*$ and $A, B \in V$.

   Similarly, G is *left-linear* if and only if every rule in R is of the form $A \rightarrow Bx$ or $A \rightarrow x$ for $x \in \Sigma^*$ and $A, B \in V$.

   Prove that if G is either right-linear or left-linear, then $L(G)$ is regular.