Lemma 2.27 (Lemma 2.15 in 1st edition) of Sipser’s text describes a general conversion from PDA’s to CFG’s. We will apply it to the following PDA which accepts the language \( \{a^n b^n \mid n \geq 0\} \) and has all the properties required for the direct construction.

For the general construction there would be 9 variables \( A_{ij} \) for \( i, j \in \{0, 1, 2\} \). However we only need to include those \( A_{ij} \) such that there is a path from \( i \) to \( j \) in the PDA diagram. This means that we only need to include rules involving \( A_{00}, A_{01}, A_{02}, A_{11}, A_{12}, A_{22} \).

The start symbol is \( A_{02} \). The rules involving only these symbols are:

\[
\begin{align*}
A_{00} & \rightarrow A_{00} A_{00} \mid \varepsilon \\
A_{01} & \rightarrow A_{00} A_{01} \mid A_{01} A_{11} \\
A_{02} & \rightarrow A_{00} A_{02} \mid A_{01} A_{12} \mid A_{02} A_{22} \\
A_{11} & \rightarrow A_{11} A_{11} \mid \varepsilon \\
A_{12} & \rightarrow A_{11} A_{12} \mid A_{12} A_{22} \\
A_{22} & \rightarrow A_{22} A_{22} \mid \varepsilon \\
A_{02} & \rightarrow a A_{02} b \\
A_{02} & \rightarrow A_{11}
\end{align*}
\]

where the next to last rule comes from pairing up the two self loop arcs and the last rule comes from pairing up the other two arcs.