Problem Set 8
Due Friday, March 11, 2005, in class

Reading assignment: Sections 3.1, 3.3, 4.1, 4.2 of Sipser’s text.
There are **FOUR** questions. Each question is worth **15 points**.

1. (a) Show that decidable languages are closed under union, intersection, and complementation.
   (b) Show that Turing-recognizable languages are closed under union and intersection.

2. Show using a proof by diagonalization that the set of all infinite sequences over \{0, 1\} is uncountable.

3. Let \( C \) be a language. Prove that \( C \) is Turing-recognizable if and only if there exists a decidable language \( D \) such that \( C = \{ x : \exists y (\langle x, y \rangle \in D) \} \).
   **Hint:** For the only if part, it might help to think of \( y \) as the witness or proof that a string \( x \) is accepted by a Turing Machine. So think of what could serve as such a witness.

4. Define the language
   \[ A = \{ \langle M \rangle \mid M \text{ is a DFA that only accepts strings over } \{0, 1\} \text{ with an odd number of 1’s} \} \).
   Show that \( A \) is decidable.
   **Suggestion:** Theorem 4.4 of Sipser’s book might be useful.