1. (15 points) For each of the following statements, answer whether they are True or False by circling the appropriate choice. You do not need to justify your answer.

(a) If $A \neq B \neq C$ are languages such that $A \cap B = C$ and $B, C$ are both regular, then $A$ must also be regular.

(b) If $L$ is regular, then so is the language $\{xy \mid x \in L, y \notin L\}$.

(c) If $L$ is regular, the minimum state DFAs for both $L$ and $\overline{L}$ have the same number of states.

(d) $b^*a^* \cap a^*b^* = a^* \cup b^*$

(e) The minimum state DFA for the language $\{w \in \{a, b\}^* \mid w \text{ contains } abaab \text{ as a substring} \}$ has more than 6 states.

2. (30 points) Define the language $A = \{w \in \{0, 1\}^* \mid \text{the number of } 0\text{'s minus the number of } 1\text{'s in } w \text{ is divisible by } 3\}$.

(a) Construct a DFA with only three states that recognizes $A$.

(b) Prove that your DFA from Part (a) is optimal, i.e. three states are the minimum needed to recognize $A$.

(c) Using the state elimination procedure described in class or otherwise, write down a regular expression that generates the language $A$.

3. (20 points) Using the pumping lemma for regular languages, prove that the language

$$\{a^nba^mba^{m+n} \mid n, m \geq 1\}$$

is not regular.

4. (15 points) Prove or disprove: If $B \subseteq \{0, 1\}^*$ is a regular language, then the language $C = \{x \in B \mid x \text{ does not contain } 1101 \text{ as a substring} \}$ is also regular.

5. (20 points) Design a context-free grammar for the language $\{0^i1^j \mid j > i \geq 1\}$. Draw a parse tree for your grammar that derives the string $0^31^4$. Is this parse tree uniquely determined for your grammar?