1. (15 points) Write formal descriptions of the following sets:
   Examples: Set containing 1, 10, 100 = \{1, 10, 100\}
   a. The set containing all even integers = \{n \mid n = 2m \text{ for some } m \in \mathbb{Z}\}
   b. The set containing all natural numbers that are greater than −5 and less than 0
   c. The set containing all integers that are odd and divisible by 3
   d. The set containing all strings (over \(\Sigma = \{0,1\}\)) that do not contain 111 as a substring
   e. The set containing all strings (over \(\Sigma = \{0,1\}\)) of odd length whose middle symbol is 0

2. (20 points) Let \(A = \{n \mid n \text{ is a prime number and } 10 < n < 20\}\) and let \(B = \{n \mid n = 2m-1 \text{ for some } m \in \mathbb{N} \text{ and } 5 < m < 10\}\).
   a. Which of the following 4 statements is/are true:
      i. \(A \subseteq B\)
      ii. \(B \subseteq A\)
      iii. \(A \cap B \neq \emptyset\)
      iv. \(A \cup B = \{n \mid n = 2m+1 \text{ for some } m \in \mathbb{N} \text{ and } 5 \leq m < 10\}\)
     v. \((B - A) = \{15,19\}\) (Recall the definition of “−” for sets discussed in class in the proof example for set equality)
   b. What is the complement of \(A \cup B?\) (Write a formal description in the form: \(\{n \mid \ldots \}\). Take complement with respect to \(\mathbb{N}\))
   c. What is \(A \times B?\) (List all the elements)
   d. Is \(A \times B = B \times A?\) Why/Why not?
   e. What is the power set of \(A \cap B\) (i.e. \(2^{A \cap B}\))? (List all the elements)

3. (15 points) Let \(A, B,\) and \(C\) be any three sets. Prove or disprove:
   \[A - (B \cap C) = (A - B) \cap (A - C)\]

4. (20 points) Prove that for any two integers \(x\) and \(y,\) \(xy\) is odd if and only if both \(x\) and \(y\) are odd.

5. (30 points) Prove the following:
   a. For any two strings \(x\) and \(y\) in \(\Sigma^*,\) \((xy)^R = y^R x^R\) (Hint: Use induction on the length of \(y\)).
   b. For any string \(x\) in \(\Sigma^*\) and any \(k \geq 0,\) \((x^k)^R = (x^R)^k\) (Hint: Use induction on \(k\)).