Beyond the Regular world…

✧ Are there languages that are not regular?

✧ **Idea:** If a language violates a property obeyed by all regular languages, it cannot be regular!
  ✧ **Pumping Lemma** for showing non-regularity of languages

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The Pumping Lemma for Regular Languages

✧ **What is it?**
  ✧ A statement (“lemma”) that is true for all regular languages

✧ **Why is it useful?**
  ✧ Can be used to show that certain languages are not regular
  ✧ How? **By contradiction:** Assume the given language is regular and show that it does not satisfy the pumping lemma
More about the Pumping Lemma

✦ **What is the idea behind it?**
  ✦ Any regular language \( L \) has a DFA \( M \) that recognizes it
  ✦ If \( M \) has \( p \) states and accepts a string of length \( \geq p \), the sequence of states \( M \) goes through must contain a cycle (repetition of a state) due to the pigeonhole principle! Thus:
  ✦ *All strings* that make \( M \) go through this cycle 0 or any number of times are also accepted by \( M \) and should be in \( L \).

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Formal Statement of the Pumping Lemma

✦ **Pumping Lemma:** If \( L \) is regular, then \( \exists p \) such that \( \forall s \) in \( L \) with \( |s| \geq p \), \( \exists x, y, z \) with \( s = xyz \) and:
  1. \( xyz \in L \) \( \forall i \geq 0 \), and
  2. \( |y| \geq 1 \), and
  3. \( |xy| \leq p \).

✦ We did the proof on board last time…(see also page 79 in textbook)

✦ Proved in 1961 by Bar-Hillel, Peries and Shamir
Pumping Lemma in Plain English

Let L be a regular language and let \( p = \text{“pumping length”} = \text{no. of states of a DFA accepting } L \)

Then, any string \( s \) in L of length \( \geq p \) can be expressed as \( s = xyz \) where:
- \( y \) is not null (\( y \) is the cycle)
- \( |xy| \leq p \) (cycle occurs within \( p \) state transitions), and
- any “pumped” string \( xy^iz \) is also in L for all \( i \geq 0 \) (go through the cycle 0 or more times)

I liked the formal statement better…

That’s more like it…

Using The Pumping Lemma

In-Class Examples: Using the pumping lemma to show a language L is not regular

5 steps for a proof by contradiction:
1. Assume L is regular.
2. Let \( p \) be the pumping length given by the pumping lemma.
3. Choose cleverly an \( s \) in L of length at least \( p \), such that
4. For all ways of decomposing \( s \) into \( xyz \), where \( |xy| \leq p \) and \( y \) is not null,
5. There is an \( i \geq 0 \) such that \( xy^iz \) is not in L.

Can’t wait to use it…
Proving non-regularity as a Two-Person game
✦ An alternate view: Think of it as a *game between you and an opponent (JC):*

1. **You:** Assume $L$ is regular
2. **JC:** Chooses some value $p$
3. **You:** Choose cleverly an $s$ in $L$ of length $\geq p$
4. **JC:** Breaks $s$ into some $xyz$, where $|xy| \leq p$ and $y$ is not null,
5. **You:** Need to choose an $i \geq 0$ such that $xy^iz$ is not in $L$ (in order to win (the prize of non-regularity)!)

(Note: Your $i$ should work for all $xyz$ that JC chooses, given your $s$)

Proving Non-Regularity using the Pumping Lemma
✦ On-Board Examples: Show the following are not regular
- $L_1 = \{0^n1^n \mid n \geq 0\}$ over the alphabet $\{0, 1\}$
- $ADD = \{x=y+z \mid x, y, z$ are binary numbers and $x$ is the sum of $y$ and $z\}$ over the alphabet $\{0, 1, =, +\}$
- $L_2 = \{0^p \mid p \text{ is prime}\}$ over the alphabet $\{0\}$
Da Pumpin’ Lemma
(Orig. lyrics: Harry Mairson)

Any regular language \( L \) has a magic numba \( p \)
And any long-enuff word \( s \) in \( L \) has da followin’ propa’ty:
Amongst its first \( p \) symbols is a segment you can find
Whose repetition or omission leaves \( s \) amongst its kind.

So if ya find a language \( L \) which fails dis acid test,
And some long word you pump becomes distinct from all da rest,
By contradiction you have shown dat language \( L \) is not
A regular homie, resilient to the damage you have wrought.

But if, upon the other hand, \( s \) stays within its \( L \),
Then either \( L \) is regular, or else you chose not well.
For \( s \) is \( xyz \), where \( y \) cannot be empty,
And \( y \) must come before da \( p+1^{th} \) symbol is read.

R. Rao, CSE 322
Based on: http://www.cs.brandeis.edu/~mairson/poems/nodel.html

If \( \{0^n1^n \mid n \geq 0\} \) is not Regular, what is it?

Enter…the world of Grammars
(after the Midterm)
Next Class: Midterm Review