1. (30 points) Closure properties:
   a. Show that decidable languages are closed under:
      i. complementation
      ii. concatenation
   b. Show that Turing-recognizable languages are closed under the union operator.
      Give implementation level details of the necessary Turing machines in each case.

2. (40 points) Exercises 4.3 and 4.4 in the textbook. (Hint: Construct your decider TMs based on deciders constructed in proofs for theorems in Section 4.1. Explain your proof idea and then give your TM in the format $M = \text{"On input \ldots:"}$).

3. (30 points) Let $ALL_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \Sigma^* \}$. Show that $ALL_{TM}$ is undecidable by giving a reduction from a known undecidable language to $ALL_{TM}$. For your reduction, you may use $A_{TM}$ or any of the languages shown to be undecidable in Section 5.1 in the textbook (pages 171-176 only).

Bonus question (no points!): Is the question “Does God exist?” decidable? (Hint: See Problem 3.19 in the textbook)