Beyond the Regular world…

✦ Are there languages that are *not* regular?

✦ **Idea:** If a language violates a property obeyed by all regular languages, it cannot be regular!

✦ **Pumping Lemma** for showing *non-regularity* of languages

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The Pumping Lemma for Regular Languages

✦ **What is it?**

   ✦ A statement ("lemma") that is true for all regular languages

✦ **Why is it useful?**

   ✦ Can be used to show that certain languages are *not regular*

   ✦ **How?** *By contradiction:* Assume the given language is regular and show that it does not satisfy the pumping lemma
More about the Pumping Lemma

- **What is the idea behind it?**
  - Any regular language $L$ has a DFA $M$ that recognizes it
  - If $M$ has $p$ states and accepts a string of length $\geq p$, the sequence of states $M$ goes through must contain a cycle (repetition of a state) due to the pigeonhole principle! Thus:
  - *All strings* that make $M$ go through this cycle 0 or any number of times are also accepted by $M$ and *should be in* $L$.

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Formal Statement of the Pumping Lemma

- **Pumping Lemma:** If $L$ is regular, then $\exists p$ such that $\forall s \in L$ with $|s| \geq p$, $\exists x, y, z$ with $s = xyz$ and:
  1. $xyz \in L \forall i \geq 0$, and
  2. $|y| \geq 1$, and
  3. $|xy| \leq p$.
- Proof on board…(see also page 79 in textbook)
- Proved in 1961 by Bar-Hillel, Peries and Shamir
Pumping Lemma in Plain English

- Let $L$ be a regular language and let $p = \text{“pumping length”} = \text{no. of states of a DFA accepting } L$
- Then, any string $s$ in $L$ of length $\geq p$ can be expressed as $s = xyz$ where:
  - $y$ is not empty ($y$ is the cycle)
  - $|xy| \leq p$ (cycle occurs within $p$ state transitions), and
  - any "pumped" string $xy^iz$ is also in $L$ for all $i \geq 0$ (go through the cycle 0 or more times)

Using The Pumping Lemma

- **In-Class Examples:** Using the pumping lemma to show a language $L$ is *not* regular
  - 5 steps for a proof by contradiction:
    1. Assume $L$ is regular.
    2. Let $p$ be the pumping length given by the pumping lemma.
    3. Choose cleverly an $s$ in $L$ of length at least $p$, such that
    4. For *all ways* of decomposing $s$ into $xyz$, where $|xy| \leq p$ and $y$ is not null,
    5. There is an $i \geq 0$ such that $xy^iz$ is not in $L$. 
Proving Non-Regularity using the Pumping Lemma

✦ Examples: Show the following are not regular
  - $L_1 = \{0^n1^n \mid n \geq 0\}$ over the alphabet $\{0, 1\}$
  - $L_2 = \{w \mid w$ contains equal number of 0s and 1s$\}$ over the alphabet $\{0, 1\}$

✦ Try these at home:
  - $L_3 = \{0^n1^m \mid n > m\}$ over the alphabet $\{0, 1\}$
  - $ADD = \{x=y+z \mid x, y, z$ are binary numbers and $x$ is the sum of $y$ and $z\}$ over the alphabet $\{0, 1, \ =, +\}$
  - $PRIMES = \{0^p \mid p$ is prime$\}$ over the alphabet $\{0\}$