Review of Proof Techniques

✦ Contents of the CSE 322 Proofs Toolbox:
  ✦ Proof by counterexample: Give an example that disproves the given statement. E.g. PRIMES ⊆ ODD
  ✦ Proof by contradiction: Assume statement is false and show that it leads to a contradiction.
  ✦ Proof by construction
  ✦ Proof of set equality A = B: Show A ⊆ B and B ⊆ A
  ✦ Proof of “X iff Y” (or X ⇔ Y) statements
  ✦ Proof by induction
  ✦ “Birdy” technique #1: Pigeonhole principle
  ✦ “Birdy” technique #2: Dovetailing
  ✦ CS Theoretician’s favorite: Diagonalization

Proof Techniques Review:
The Big picture

✦ Proof by contradiction: Assume statement is false and show that it leads to a contradiction
  ✦ E.g.: Prove: Complement of any finite subset of Z is infinite

✦ Proof by construction: Show that a statement can be satisfied by constructing an object using what is given
  ✦ E.g.: Show that for all c, ∃ n₀ s.t. n² > cn for all n ≥ n₀

✦ Proof of set equality A = B: Show A ⊆ B and B ⊆ A
  ✦ E.g.: De Morgan’s Law (one of two):
    A – (B ∪ C) = (A – B) ∩ (A – C)

✦ Proving “X iff Y” statements: Prove X ⇒ Y (“X only if Y”) and Y ⇒ X (“X if Y”)
  ✦ E.g.: For all real numbers x, show ⌊x⌋ = [x] iff x ∈ Z
Review: Avian Technique #1

- **Pigeonhole principle:** If A and B are finite sets and \(|A| > |B|\), then there is no one-to-one function from A to B
  - \(f : A \rightarrow B\) is one-to-one if for any distinct \(x, y \in A\), \(f(x) \neq f(y)\)
  - **Idea:** “more pigeons than pigeonholes” at least one pigeonhole contains two pigeons.
  - E.g. In a room of 13 or more people, at least 2 have same birthmonth
  - Proof? By induction on \(|B|\)

- What is “Proof by Induction”?

Proof by Induction

- **Proof by induction** (very common in CS Theory): 2 steps –
  1. **Basis Step:** Show statement is true for some finite value \(n_0\), typically \(n_0 = 0\) or 1
  2. **Induction Hypothesis and Induction Step:**
     Assume statement is true for some fixed but arbitrary \(k \geq n_0\). Show it is also true for \(k + 1\)

- **Example:** Show that for all \(n \geq 1\), \(1 + 2 + \ldots + n = n(n+1)/2\)
To Infinity and Beyond  (with apologies to Disney)

✦ Sizing up sets: Cardinality of a set and countably infinite sets

✦ Avian Technique #2 – Dovetailing: Useful for showing union of any finite or countably infinite collection of countably infinite sets is again countably infinite
  ➔ Set A is countably infinite if there is a 1-1 correspondence ("bijection") between N (the set of natural numbers) and A
  ➔ E.g. Use dovetailing to show Z and N x N are both countably infinite
  ➔ A set is uncountable if it is neither finite nor countably infinite

✦ Diagonalization and Uncountable Sets: See pages 160-163 in the text for a nice introduction and more examples.
  ➔ E.g.: Set of real numbers in the interval (0,1) is uncountable

✦ See Handout #1 for more details…

Are we done with this review yet?

Enter…the finite automaton…